

# Framework based on stochastic L-Systems for modeling IP traffic with multifractal behavior

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## ABSTRACT

In a previous work we have introduced a multifractal traffic model based on so-called stochastic L-Systems, which were introduced by biologist A. Lindenmayer as a method to model plant growth. L-Systems are string rewriting techniques, characterized by an alphabet, an axiom (initial string) and a set of production rules. In this paper, we propose a novel traffic model, and an associated parameter fitting procedure, which describes jointly the packet arrival and the packet size processes. The packet arrival process is modeled through a L-System, where the alphabet elements are packet arrival rates. The packet size process is modeled through a set of discrete distributions (of packet sizes), one for each arrival rate. In this way the model is able to capture correlations between arrivals and sizes. We applied the model to measured traffic data: the well-known pOct Bellcore, a trace of aggregate WAN traffic and two traces of specific applications (Kazaa and Operation Flashing Point). We assess the multifractality of these traces using Linear Multiscale Diagrams. The suitability of the traffic model is evaluated by comparing the empirical and fitted probability mass and autocovariance functions; we also compare the packet loss ratio and average packet delay obtained with the measured traces and with traces generated from the fitted model. Our results show that our L-System based traffic model can achieve very good fitting performance in terms of first and second order statistics and queuing behavior.

**Keywords:** Traffic modeling, Multifractal, L-System.

## 1. INTRODUCTION

Accurate modeling of IP traffic requires the characterization of both the packet arrival and the packet size processes. In particular, this is important for accurate prediction of the queuing behavior (i.e., the packet loss ratio or average packet delay suffered on a network node). The queuing behavior addresses the effect of traffic on network performance, and is one of the most important criteria to assess the suitability of traffic models (and associated parameter fitting procedures). Here, the analysis consists in comparing the curves of packet loss ratio (or average packet delay) versus buffer size, obtained with the measured traces (through trace-driven simulation) and with the inferred traffic model (using again trace-driven simulation or numerical computation of the performance measures whenever possible). When dealing with models that characterize only the arrival process, it is common practice to assume that the packet size is fixed and equal to the average packet size of the measured trace. This may lead to large errors when the packets have variable size, such as in IP traffic.

Recent analysis of measured Internet WAN traffic has revealed that multifractal structures, such as random cascades, can help explaining the scaling behavior typically associated to networking mechanisms operating on small time scales (e.g. TCP flow control). A cascade (or multiplicative process) is a process that fragments a set into smaller and smaller components according to a fixed rule, and at the same time fragments the measure of the components by another (possibly random) rule. Random cascades were introduced by Mandelbrot as a physical model for turbulence.<sup>1</sup> In the traffic modeling context, the set can be interpreted as a time interval and the measure as the number of arrivals or number of bytes (in that interval).

The multifractal nature of network traffic was first noticed by Riedi and Lévy Véhel.<sup>2</sup> Subsequently various studies have addressed the characterization and modeling of multifractal traffic, essentially within the framework of random cascades.<sup>3–10</sup> Feldmann *et al.*<sup>38</sup> proposed conservative cascades, which are closely related with random cascades, as a model

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for Internet WAN traffic. They provided plausible physical explanations supporting the adoption conservative cascades and developed tools to analyze the scaling behavior of this type of cascades. Gao and Rubin considered two extensions of the conservative cascade model: they introduced a dual cascade model, where one cascade characterizes packet inter-arrivals and the other packet sizes<sup>7</sup>; moreover, they considered a cascade model for the counting process representing the aggregation of the packet arrival and packet size processes, i.e., the number of bytes arriving in every time interval.<sup>11</sup> The latter model precludes estimating the packet loss ratio, since the detail of packet sizes is lost through aggregation.

In this paper, we propose a novel multifractal traffic model, and an associated fitting procedure, that characterizes both the packet arrival and packet size processes. The traffic model is based on stochastic Lindenmayer-Systems (hereafter referred to as L-Systems). L-Systems are string rewriting techniques which were introduced by biologist A. Lindenmayer in 1968 as a method to model plant growth.<sup>12</sup> They are characterized by an alphabet, an axiom and a set of production rules. The alphabet is a set of symbols; the production rules define transformations of symbols into strings of symbols; starting from an initial string (the axiom), an L-System constructs iteratively sequences of symbols replacing each symbol by the corresponding string according to the production rules. Stochastic L-Systems are a method to construct recursively random sequences with multifractal behavior.<sup>13</sup> When compared with conservative cascades, stochastic L-Systems introduce a dependence on the construction process (due to the production rules), that has a meaningful physical explanation, and can help understanding the joint impact of network mechanisms and resource limitations on observed traffic.

In the proposed traffic model only the packet arrival process is modeled through an L-System. Here, the alphabet is a set of packet arrival rates (number of arrivals per time interval, in pkts/sec) and the arrival rates are associated with time intervals; the production rules randomly generate two arrival rates from a single one; thus, starting from an initial arrival rate and time interval (at the coarsest time scale), in each iteration of the L-System construction the number of arrival rates is duplicated and the width of the associated time intervals halved, generating successively finer time scales. The characterization of the packet size is performed by associating, at the finest time scale, a discrete distribution (of packet sizes) to each packet arrival rate. In this way the model is able to capture correlations between packet arrivals and packet sizes. Moreover, the model also includes the ability of capturing multiple scaling behavior.

The application of stochastic L-Systems in the traffic modeling context was first introduced by the authors.<sup>14</sup> This first work only addressed the modeling of the packet arrival process. Two subsequent extensions addressed the modeling of the packet arrival and packet size processes. The first extension defined a model with two independent L-Systems, one for packet arrivals and the other for packet sizes,<sup>15</sup> called double L-System. In the second L-System the alphabet elements are mean packet sizes, where the mean is calculated over the sampling interval. Due to the independence of the L-Systems, this model does not capture the correlations between arrivals and sizes (although it captures multifractal behavior on both arrivals and sizes). The second extension defined a bi-dimensional L-System, called joint L-System, where the alphabet elements are now pairs of arrival rates and mean packet sizes.<sup>16</sup> Opposite to the previous model, this one is able to capture correlations between arrivals and sizes. One disadvantage, however, is that it may require a large number of parameters. The model presented in this paper was devised in order to allow a lower number of parameters and also to provide a more detailed modeling of the packet size. Note that in the proposed model the packet sizes are modeled individually, whereas in the two previous models only the mean packet sizes were modeled. However, the proposed model does not capture multifractal behavior on the packet sizes.

This paper is organized as follows. In section 2 we give some background on L-Systems. In sections 3 and 4 we present the traffic model and describe the associated fitting procedure. In section 5 we discuss the results of applying the proposed fitting procedure to measured traffic traces. Finally, section 6 presents the conclusions.

## 2. L-SYSTEMS BACKGROUND

The basic idea behind L-Systems is to define complex objects by successively replacing parts of a simple object using a set of rules. The L-System is a feedback machine that operates on strings of symbols. The set of symbols is called the alphabet. Starting from an initial state (called axiom), an L-System operates, at each iteration, by applying the set of production (or rewriting) rules simultaneously to all symbols of an input string to give an output string. For a comprehensive introduction to L-Systems see the book by Peitgen *et al.*<sup>13</sup>

Consider a simple example of an organism growing through cell subdivisions. There are two types of cells represented by letters A and B. Cell subdivisions are modeled by replacing these symbols with strings of symbols: cell A subdivides into two cells represented by string AB; cell B subdivides into two cells represented by string AA. The ordering of the

symbols is relevant in an L-System. The organism modeled by this L-System grows by repeated cell subdivisions. At birth the organism is the single cell A. After one subdivision the organism is two cells represented by string AB. After two subdivisions, the organism has four cells given by string ABAA, and after three subdivisions the organism has eight cells represented by string ABAAABAB. Using the formalism of L-Systems this growth process can be described as:

Alphabet:  $\{A,B\}$   
Axiom: A  
Rules:  $A \rightarrow AB$   
 $B \rightarrow AA$

The production rules can be stochastic. In stochastic L-Systems there may be several production rules for one symbol, and the specific rule is selected according to a probability distribution. Taking previous example, one production rule could be to convert A into AB with probability 0.4 or into BB with probability 0.6 (instead of converting always A into AB). In this case, after 3 iterations several strings are possible, e.g., ABAAABAB, ABABBBAB, or AAAAABBB. Stochastic L-Systems are a method to construct recursively random sequences with multifractal behavior.<sup>13</sup>

As another example, consider the following stochastic L-System:

Alphabet:  $\{X_1, X_2, X_3, X_4, X_5\}$   
Axiom:  $X_3$   
Rule 1:  $X_1 \rightarrow X_1X_1$   
Rule 2: (with prob. 1/3)  $X_2 \rightarrow X_1X_3$   
(with prob. 1/3)  $X_2 \rightarrow X_3X_1$   
(with prob. 1/3)  $X_2 \rightarrow X_2X_2$   
Rule 3: (with prob. 1/5)  $X_3 \rightarrow X_3X_3$   
(with prob. 1/5)  $X_3 \rightarrow X_2X_4$   
(with prob. 1/5)  $X_3 \rightarrow X_4X_2$   
(with prob. 1/5)  $X_3 \rightarrow X_1X_5$   
(with prob. 1/5)  $X_3 \rightarrow X_5X_1$   
Rule 4: (with prob. 1/3)  $X_2 \rightarrow X_3X_5$   
(with prob. 1/3)  $X_4 \rightarrow X_5X_3$   
(with prob. 1/3)  $X_4 \rightarrow X_4X_4$   
Rule 5:  $X_5 \rightarrow X_5X_5$

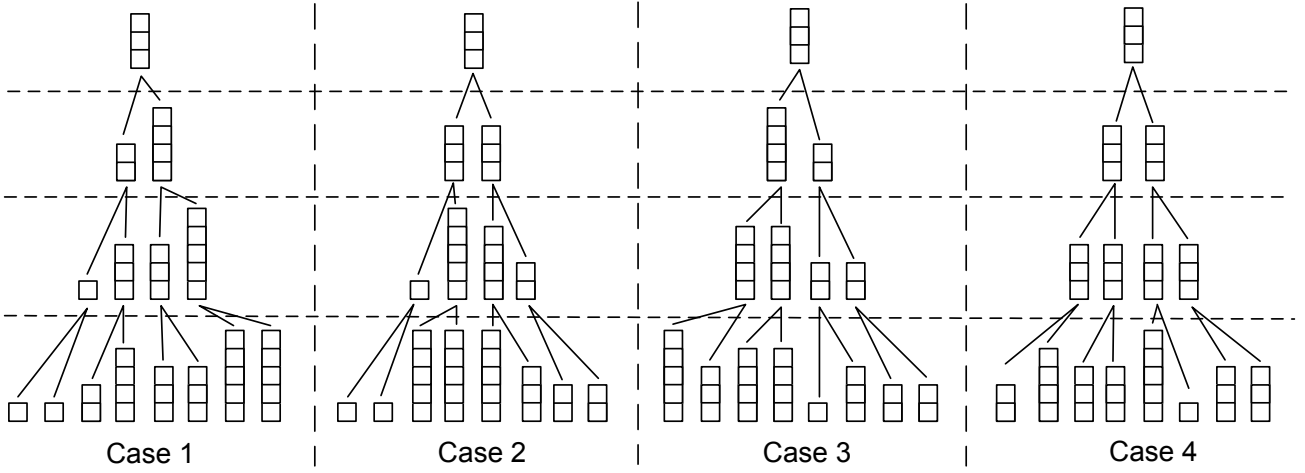
The alphabet elements of this L-System can be associated to graphical elements. For example,  $X_i$  can represent a rectangle with an area of  $i$  units. The L-System production rules assure that the average area of the child rectangles is equal to the area of its parent rectangle. Some of the possible outcomes of the L-System construction are represented in Figure 1.

In order to build a network traffic model, the area of a rectangle can be interpreted as the amount of traffic (number of arrivals or number of bytes, observed in a particular time interval. This is precisely the idea behind the proposed traffic model, that will be described in the following section.

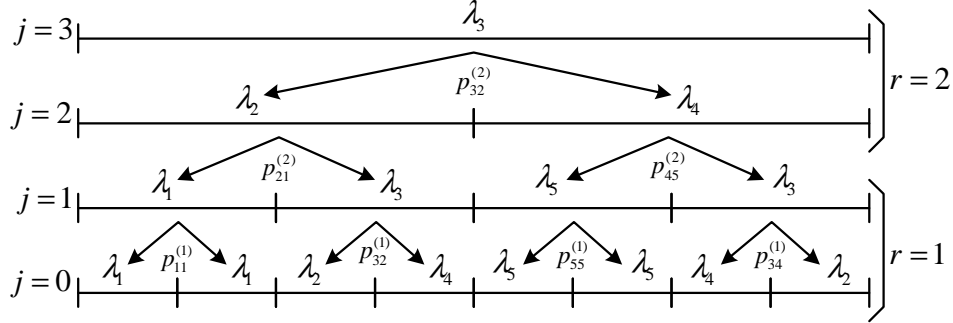
### 3. TRAFFIC MODEL

The traffic model introduced in this section consists in a L-System to characterize the packet arrival process and a set of discrete distributions to characterize the packet size (one for each packet arrival rate). In this way, the model is able to capture correlations between packet arrivals and packet sizes, as well as multifractal behavior on the packet arrival process. We will call this model L-System with PMFs, due to the fact that the L-System will be associated to a set of probability mass functions for modeling the packet size process.

We start by describing the L-System modeling the packet arrival process. We work on an alphabet of arrival rates defined by



**Figure 1.** Example of stochastic L-System with rectangles.



**Figure 2.** Construction of the packet arrival process based on a L-System.

$$\Lambda = \{\lambda_1, \lambda_2, \dots, \lambda_L\}, \lambda_i \in \mathbb{R}_0^+, i = 1, \dots, L. \quad (1)$$

and with production rules that randomly generates two arrival rates from a previous one. Without loss of generality, we assume  $\lambda_1 < \lambda_2 < \dots < \lambda_L$ .

The packet arrival process is constructed progressively, governed by an L-System machine, where each iteration produces a new time scale. Starting with the coarsest time scale, where traffic is characterized by a single arrival rate over a single time interval, each iteration generates a finer time scale by (i) division of each (parent) time interval in two new equal length (child) subintervals and (ii) association of arrival rates to each new subinterval according to the production rules of the stochastic L-System. We allow the grouping of time scales in time scale ranges and the definition of different sets of production rules for each time scale range. This is motivated by the fact that each set of production rules maps into a distinct scaling behavior.<sup>17</sup> Classically, there will be scaling if the log-log plot of the  $q^{th}$  order energies (usual energy is  $q = 2$ ) as a function of scale behaves linearly; if the plot is (globally) non-linear, different time scale ranges can be detected where linearity is observed (see, for example, the work by Abry *et al.*<sup>6</sup>). The packet arrival process construction is illustrated in Figure 2.

To characterize the packet arrival process we define  $X_{(j,r)}^{(i)} \in \Lambda$  as the arrival rate at time interval  $i$  of time scale  $j$  and time scale range  $r$ . Let the number of scales be  $S$  and the number of ranges of scales be  $R$ . For convenience, we let  $j$  decrease from  $j = S - 1$  (at the coarsest time scale) to  $j = 0$  (at the finest time scale). Also, we let  $r$  decrease from  $r = R$  (the range of coarsest time scales) to  $r = 1$  (the range of finest time scales). Thus, the number of time intervals at time scale  $j$ , which we will denote by  $N_j$ , is  $2^{S-j-1}$ . Moreover, assuming a unitary width for the intervals of the finest time

scale,  $j = 0$ , the width at scale  $j$  will be  $2^j$ . To relate time scales and time scale ranges we define  $j_r$  as the coarsest scale  $j$  in range  $r$ . Thus, in Figure 2,  $S = 4$ ,  $R = 2$ ,  $j_2 = 3$  and  $j_1 = 1$ .

In order to assure that the average arrival rate is the same in all time scales, so as to maintain physical meaning, we will impose the following condition to the production rules:

$$X_{(j,r)}^{(i)} = \frac{1}{2}X_{(j-1,r')}^{(2i-1)} + \frac{1}{2}X_{(j-1,r'')}^{(2i)} \quad (2)$$

i.e., the mapping of arrival rates is such that the arrival rate averaged over the left and right child subintervals will be equal to the parent arrival rate. With this condition, the traffic process generation can be described by axiom  $X_{(S-1,R)}^{(1)}$ , the arrival rate at the coarsest time scale, and production rules defined by

$$X_{(j,r)}^{(i)} = \lambda_l \xrightarrow{p_{lq}^{(r)}} \begin{cases} X_{(j-1,r')}^{(2i-1)} = \lambda_q \\ X_{(j-1,r'')}^{(2i)} = 2\lambda_l - \lambda_q \end{cases} \quad (3)$$

where  $\sum_{q=1}^L p_{lq}^{(r)} = 1, \forall l$ . Thus, an arrival rate  $\lambda_l$  in interval  $i$ , scale  $j$  and range  $r$  produces, with probability  $p_{lq}^{(r)}$ , arrival rate  $\lambda_q$  at the left subinterval  $2i - 1$  and arrival rate  $2\lambda_l - \lambda_q$  at the right subinterval  $2i$ , of next scale  $j - 1$  and range  $r'$ . The production rules can be totally described by  $R \times L$  matrices

$$\mathbf{P}^{(r)} = (p_{lq}^{(r)}), \quad l, q = 1, \dots, L, \quad r = 1, \dots, R \quad (4)$$

In order to guarantee that the alphabet is closed with respect to the production rules we impose the following conditions: (i)  $\lambda_i - \lambda_{i-1} = \frac{\lambda_L - \lambda_1}{L-1}, i = 2, 3, \dots, L$ , i.e., the  $\lambda_i$  values will be equidistant; (ii)  $p_{lq}^{(r)} = 0$  if  $q > l + \min(L - l, l - 1)$  or  $q < l - \min(L - l, l - 1)$ .

Finally, the L-System construction defines, at scale  $j$  and range  $r$ , the sequence of arrival rates

$$Y_{(j,r)} = \{X_{(j,r)}^{(i)}, i = 1, \dots, N_j\} \quad (5)$$

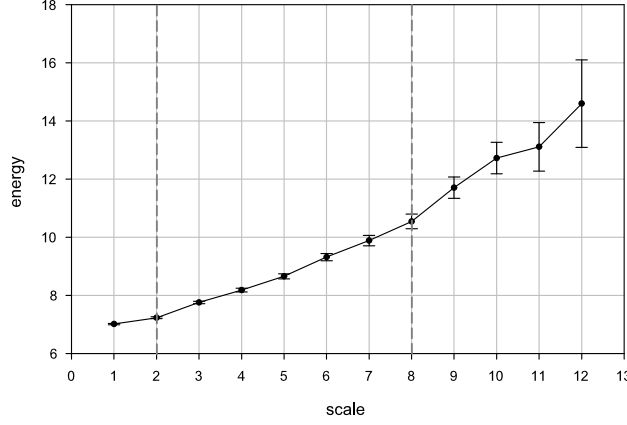
To characterize the packet size process, let  $Y$  be a discrete random variable representing the packet size. We define  $L$  PMFs for the packet sizes (one for each packet arrival rate), given by  $h^l = P(Y = y_i)$ , with  $y_i \in \Upsilon^l = \{y_1, y_2, \dots, y_{G_l}\}$  and  $l = 1, 2, \dots, L$ ;  $\Upsilon^l$  is the set of  $G_l$  packet sizes associated with arrival rate  $\lambda_l$ . Note that all packets arriving in a time interval with associated arrival rate  $\lambda_l$  will be assigned a packet size according to PMF  $h^l$ .

The generation of the complete traffic process has three steps: (i) we first generate the sequence of arrival rates at the finest time scale given by the corresponding L-System; (ii) second, within each time interval, the packet arrival instants are spaced uniformly (the number of arrivals is set according to the arrival rate of the interval); (iii) at last, the sizes of all packets are determined, based on PMFs of the associated arrival rates.

#### 4. FITTING PROCEDURE

The fitting procedure determines the parameters of the L-System describing the packet arrival process and infers the PMFs describing the packet size process, from real data observations. It starts by fixing a sampling interval  $\Delta$  and considering the time series representing the total number of packet arrivals in each non-overlapping sampling interval. Let this (empirical) time series be  $\{A_k, k = 1, 2, \dots, K\}$ , where  $A_k$  represents the number of arrivals in sampling interval  $k$ . For convenience, we take the length of the time series  $K$  to be a power of 2. In addition, we collect in set  $\Sigma$  the packet sizes of all packets contained in the  $K$  sampling intervals. We let the total number of packets (in the  $K$  sampling intervals) be  $N$ .

The inference procedure can then be divided in four steps: (i) determination of the L-System alphabet and axiom, (ii) identification of the time scale ranges, (iii) inference of the L-System production rules and (iv) inference of the packet size distributions.



**Figure 3.** Second-order logscale diagram, trace UA.

The alphabet of the L-System will consist in  $L$  equidistant arrival rate values, ranging from the minimum to the maximum values present in data. The value of  $L$  is a compromise between accuracy and complexity. Due to the mass preservation property of L-Systems defined in (2), the axiom is inferred as the average arrival rate of  $\{A_k\}$ , rounded to the closest alphabet element, i.e.,

$$X_{(S-1,R)}^{(1)} = \Gamma_{\Lambda} \left( (1/K\Delta) \sum_{k=1}^K A_k \right) \quad (6)$$

where  $\Gamma_{\Omega}(x)$  represents a function that rounds  $x$  towards the nearest element of  $\Omega$ .

The identification of time scale ranges is based on wavelet scaling analysis. We use the method proposed by Abry *et al.*,<sup>6</sup> which resorts to the (second-order) logscale diagram. A (second-order) logscale diagram is a plot of energies against scale  $j$ , together with confidence intervals about the energies, where these values are a function of the wavelet discrete transform coefficients at scale  $j$ . The time scale ranges correspond to the set of time scales for which, within the limits of the confidence intervals, the energies fall on a straight line, i.e., the scaling behavior is linear in a time scale range. Figure 3 shows the logscale diagram of a trace measured at the University of Aveiro (which is described in section 5). There are 3 time scale ranges (within a total of 18 time scales) defined by  $j_1 = 2, j_2 = 8$  and  $j_3 = 14$ .

The next step is the inference of the L-System production rules, which are fully characterized by the  $\mathbf{P}^{(s)}$  matrices. First, data is rounded in order to define sequence  $Y_{(j,r)}$  at each time scale. This comprises obtaining the arrival rates  $X_{(j,r)}^{(i)}$  from  $\{A_k\}$  through

$$X_{(j,r)}^{(i)} = \Gamma_{\Lambda} \left( (N_j/K\Delta) \sum_{k=K(i-1)/N_j+1}^{Ki/N_j} A_k \right) \quad (7)$$

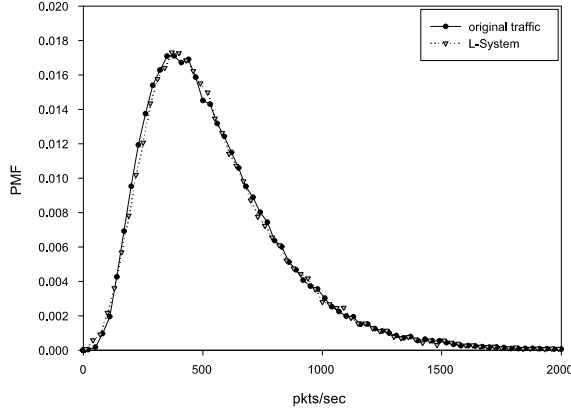
with  $i = 1, \dots, N_j$ , for each  $j$ . Letting  $c_{lq}^{(r)}$  represent the number of times that, at scale  $j$  and range  $r$ , the parent  $X_{(j,r)}^{(i)} = \lambda_l$  produced the left child  $X_{(j-1,r')}^{(2i-1)} = \lambda_q$ , the production rule probabilities can be inferred as

$$p_{lq}^{(r)} = c_{lq}^{(r)} / \sum_{q=1}^L c_{lq}^{(r)}, \quad l = 1, \dots, L, \quad r = 1, \dots, R \quad (8)$$

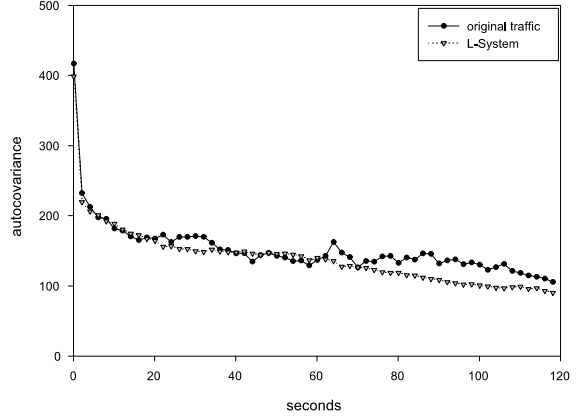
We now describe the inference process of the PMFs describing the packet size process. We first determine the  $L$  partitions of set  $\Sigma$ ,  $\Sigma^l, l = 1, 2, \dots, L$ , where  $\Sigma^l$  collects the packet sizes of all packets with associated arrival rate  $\lambda_l$ .

Trace name	Capture period	Trace size (pkts)	Mean rate (byte/s)	Mean pkt size (bytes)
pOct	Bellcore trace	1 million	362750	568
UA	12.41pm to 14.27pm, July 6 <sup>th</sup> 2001	7 millions	654780	600
Kazaa	10.26pm to 12.13pm, October 18 <sup>th</sup> 2002	1 million	194670	1225
OFP	10.26pm to 12.02pm, October 18 <sup>th</sup> 2002	0.5 million	72552	803

**Table 1.** Main characteristics of measured traces.



**Figure 4.** Probability mass function, trace pOct.



**Figure 5.** Autocovariance function, trace pOct.

Note that  $\Sigma^l \subseteq \Sigma$ ,  $\Sigma^i \cap \Sigma^j = \emptyset$  if  $i \neq j$  and  $\bigcup_{l=1,2,\dots,L} \Sigma^l = \Sigma$ . Then, to limit the number of packet sizes, we determine the  $G_l$  most frequent packet sizes of  $\Sigma^l$  (e.g. using histograms), list them in set  $\Psi^l$  and determine the sets  $\Upsilon^l$  as

$$\Upsilon^l = \Gamma_{\Psi^l}(\Sigma^l), l = 1, 2, \dots, L \quad (9)$$

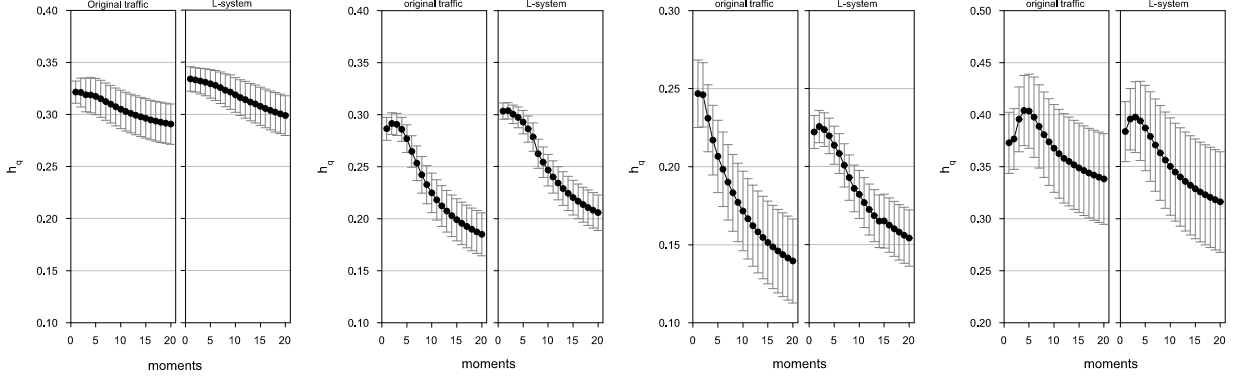
Thus  $\Upsilon^l$  contains the same elements of  $\Sigma^l$  but rounded to the nearest element in  $\Psi^l$ .

Finally we infer, for each set  $\Upsilon^l$  the corresponding PMF  $h^l$ .

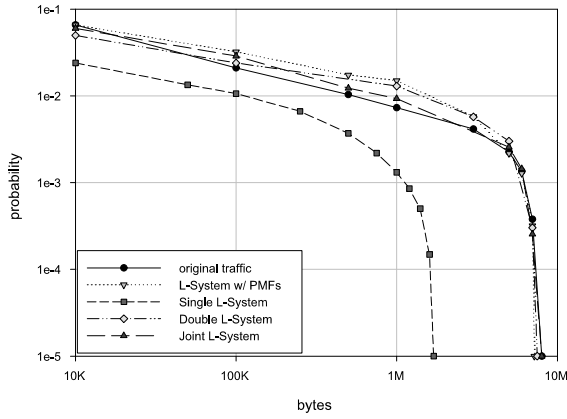
## 5. NUMERICAL RESULTS

We have applied our fitting procedure to four traces of IP traffic: (i) the well known pOct Bellcore trace, (ii) one trace measured at the University of Aveiro (UA) which exhibits non-trivial multifractal scaling behavior both on packet arrivals and packet sizes, (iii) one trace measured at a portuguese ADSL ISP, representing the usage of Kazaa by a group of 10 users and (iv) a trace measured in the same ISP, representing the usage of the online game Operation Flashing Point (OFP) also by a group of 10 users. The sampling interval was 0.1 seconds in all traces. The UA trace is representative of Internet access traffic produced within a University campus environment. The University of Aveiro is connected to the Internet through a 10 Mb/s ATM link and the measurements were carried out in a 100 Mb/s Ethernet link connecting the border router to the firewall, which only transports Internet access traffic. The traffic analyzer was a 1.2 GHz AMD Athlon PC, with 1.5 Gbytes of RAM, running WinDump. The measurements recorded the arrival instant and the IP header of each packet. WinDump reported no packet drops during the measurement periods. The main characteristics of the used traces are summarized in table 1.

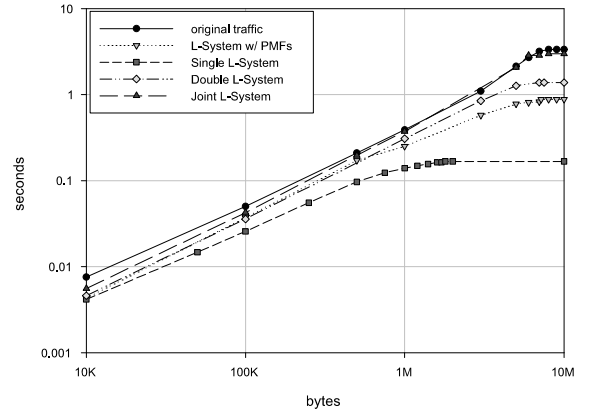
The pOct trace was fitted to an alphabet of  $L = 243$  arrival rates. The minimum and maximum arrival rates present in data were 10 and 2430 pkts/sec, respectively, and a step of 10 pkts/sec was used (because it corresponds to one arrival in the sampling interval). The logscale diagram identified 4 time scale ranges (within a total of 14 time scales) defined by  $j_1 = 5, j_2 = 8, j_3 = 9$  and  $j_4 = 13$ . In the case of the UA trace the fitted alphabet had  $L = 469$  arrival rates, ranging from



**Figure 6.** Linear multiscale diagrams, (from left to right) trace pOct, trace UA, trace Kazaa e trace OFP.



**Figure 7.** Packet loss ratio versus buffer size, trace pOct.



**Figure 8:** Average packet delay versus buffer size, trace pOct.

170 pkts/sec to 4850 pkts/sec, in steps of 10 pkts/sec. As referred before, the UA trace has 3 time scale ranges (Figure 3). The Kazaa and OFP traces were fitted to L-Systems with alphabets of  $L = 32$  arrival rates. These traces revealed the same 2 time scale ranges, defined by  $j_1 = 5$  and  $j_4 = 13$ . In all traces, the  $L$  PMFs describing the packet sizes were fitted considering, for each arrival rate, the 20 most frequent packet sizes.

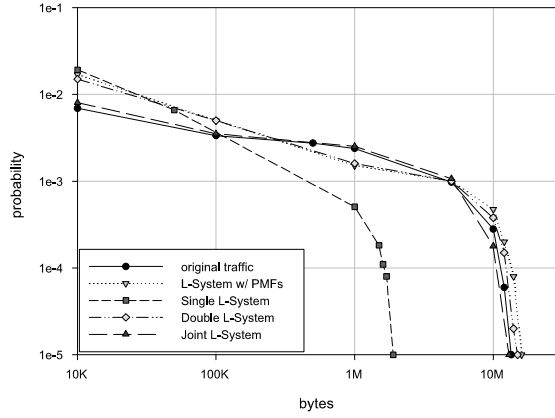
The parameter estimation took less than 2 minutes, using a MATLAB implementation running in the PC described above. This shows that the fitting procedure is computationally very efficient (note that the size of the alphabet, the number of ranges and the size of the trace, which determine the computational time, are all relatively large).

We assess the suitability of the traffic model and the accuracy of the fitting procedure using several criteria. We compare both the probability mass and autocovariance functions of the packet arrival process obtained with the fitted stochastic L-System and with the original data trace. Figures 4 and 5, reveal that, for the pOct trace, the model was able to capture the first and second order statistics with extreme precision. Similar results were obtained for other traces.

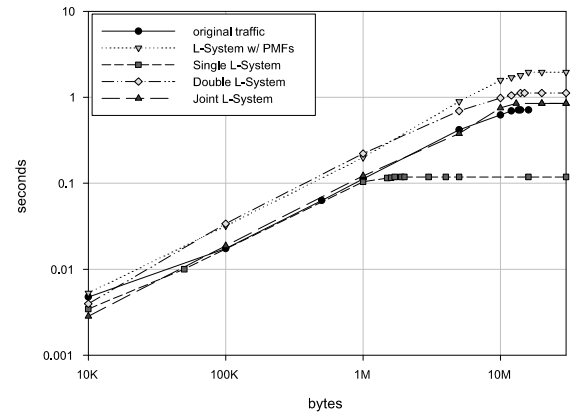
We also analyze the queuing behavior by comparing the packet loss ratio and the average packet delay, obtained through trace-driven simulation, using two types of input traffic: (i) the original traces and (ii) traces generated according to the fitted models. For the first two traces, our comparisons are extended to three other models: a single L-System with fixed packet size (equal to the mean packet size), a double L-System and a joint L-System. These models were inferred according to the methods derived by Salvador *et al.*<sup>14–16</sup>

The multifractality of the packet arrival process is assessed using a linear multiscale diagram,<sup>6</sup> which plots  $h_q = \alpha_q/q$

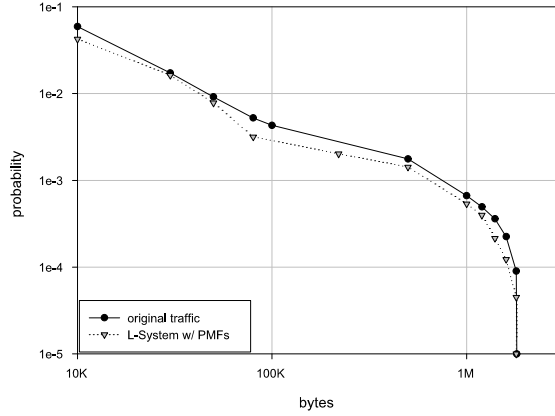




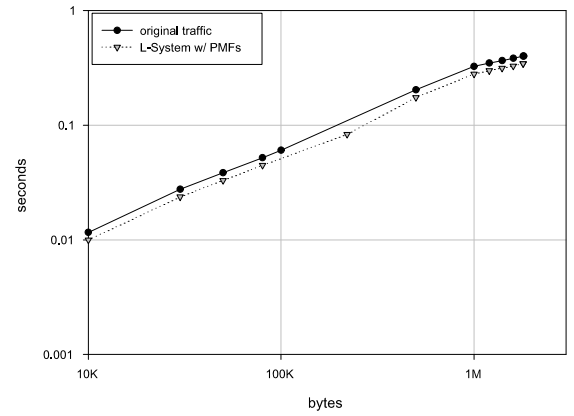
**Figure 9.** Packet loss ratio versus buffer size, trace UA.



**Figure 10:** Average packet delay versus buffer size, trace UA.



**Figure 11.** Packet loss ratio versus buffer size, trace Kazaa.

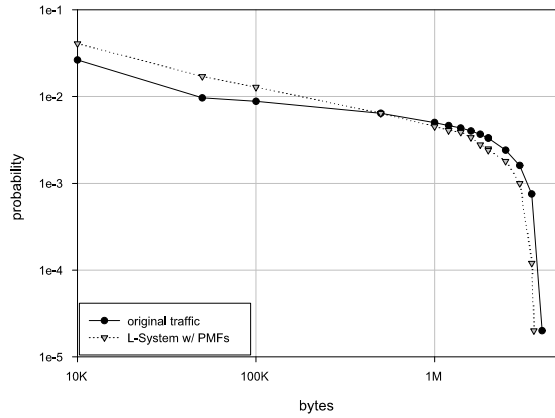


**Figure 12:** Average packet delay versus buffer size, trace Kazaa.

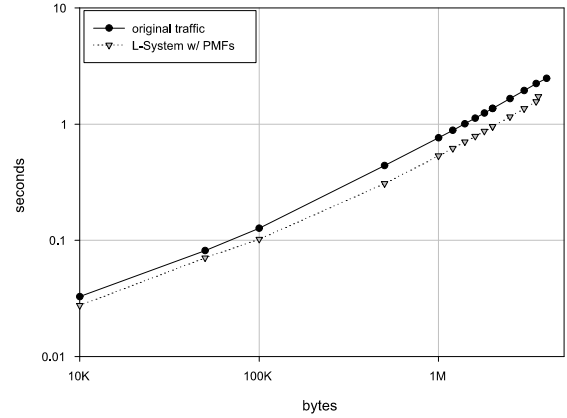
against  $q$ ;  $\alpha_q$  represents the  $q^{th}$  order scaling exponent, estimated in the  $q^{th}$  order logscale diagram. Multifractal scaling behavior is detected when there is no horizontal alignment (within the limits of confidence intervals). Figure 6 shows that the packet arrival processes of the UA and Kazaa traces, and of the traces generated using the corresponding L-Systems with PMFs, all have multifractal scaling behavior. The pOct trace is not multifractal and the OFP trace is in the limit between non-trivial and trivial multifractality. For all traces, and independently of the presence of multifractal scaling behavior, the L-System with PMFs was able to capture the scaling behavior of the original traces, since the curves of the original traces are very similar to the ones of the corresponding fitted traces.

To assess the queuing behavior the buffer size was varied from 10 Kbytes to 20 Mbytes. The service rate was 518 Kbytes/s for the pOct trace (corresponding to an utilization of 0.7) and 726 Kbytes/s for the UA trace (corresponding to an utilization of 0.9). Figures 7, 8, 9 and 10 show that, for all traces, the fitting of the queuing behavior was very good for the joint L-System model but slight differences occurred with the other L-System based models, including the L-System with PMFs presented in this paper. We also have analyzed the queuing behavior of Kazaa and OFP traces (for utilizations of 0.7), figures 11, 12, 13 and 14. The results in these cases also show that the L-System with PMFs attain a good matching of the queuing behavior.

In general, we conclude that the L-System with PMFs achieves good fitting performance, which can be partially attributed to its ability of capturing the correlations between sizes and arrivals, as well as, the multifractal behavior of the



**Figure 13.** Packet loss ratio versus buffer size, trace OFF.



**Figure 14:** Average packet delay versus buffer size, trace OFF.

packet arrival process. The performance is slightly inferior to a joint L-System which also captures multifractal behavior of the packet process. However, the L-System with PMFs has a lower number of parameters. The number of parameters of the L-System with PMFs is  $L^2 R + 20L$  and the one of the joint L-System is  $20^2 L^2 R$ , if one assumes that the packet size process is modeled with 20 sizes.

## 6. CONCLUSIONS

In this paper, we proposed a novel traffic model, and an associated parameter fitting procedure, which describes jointly the packet arrival and the packet size processes. The packet arrival process is modeled through a L-System, where the alphabet elements are packet arrival rates. The packet size process is modeled through a set of discrete distributions (of packet sizes), one for each arrival rate. In this way the model is able to capture correlations between arrivals and sizes. We applied the model to measured traffic data: the well-known pOct Bellcore, a trace of aggregate WAN traffic and two traces of specific applications (Kazaa and Operation Flashing Point). We assessed the multifractality of these traces using Linear Multiscale Diagrams. The suitability of the traffic model was evaluated by comparing the empirical and fitted probability and autocovariance functions; we also compared the packet loss ratio and average packet delay obtained with the measured traces and with traces generated from the fitted model. Our results showed that our L-System based traffic model can achieve very good fitting performance in terms of first and second order statistics and queuing behavior.

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## REFERENCES

1. B. Mandelbrot, "Intermittant turbulence in self-similar cascades: Divergence of high moments and dimensions of the carrier," *Journal of Fluid Mechanics* **62**, pp. 331–358, 1974.
2. R. Riedi and J. L. Véhel, "Multifractal properties of TCP traffic: a numerical study," *Technical Report No 3129, INRIA Rocquencourt, France*, Feb. 1997. Available at [www.dsp.rice.edu/~riedi](http://www.dsp.rice.edu/~riedi).
3. A. Feldmann, A. Gilbert, and W. Willinger, "Data networks as cascades: Investigating the multifractal nature of internet WAN traffic," in *Proceedings of SIGCOMM*, pp. 42–55, 1998.
4. R. Riedi, M. Crouse, V. Ribeiro, and R. Baraniuk, "A multifractal wavelet model with application to network traffic," *IEEE Transactions on Information Theory* **45**, pp. 992–1018, Apr 1999.

5. A. Feldmann, A. Gilbert, P. Huang, and W. Willinger, "Dynamics of IP traffic: A study of the role of variability and the impact of control," in *SIGCOMM*, pp. 301–313, 1999.
6. P. Abry, P. Flandrin, M. Taqqu, and D. Veitch, "Wavelets for the analysis, estimation and synthesis of scaling data," in *Self-Similar Network Traffic Analysis and Performance Evaluation*, K. Park and W. Willinger, Eds. , 1999.
7. J. Gao and I. Rubin, "Multifractal analysis and modeling of long-range-dependent traffic," in *Proceedings ICC'99*, pp. 382–386, June 1999.
8. A. Gilbert, W. Willinger, and A. Feldmann, "Scaling analysis of conservative cascades, with applications to network traffic," *IEEE Transactions on Information Theory* **45**, pp. 971–992, April 1999.
9. A. Erramilli, O. Narayan, A. Neidhardt, and I. Saniee, "Performance impacts of multi-scaling in wide area TCP/IP traffic," in *Proceedings of INFOCOM'2000*, 2000.
10. D. Veitch, P. Abry, P. Flandrin, and P. Chainais, "Infinitely divisible cascade analysis of network traffic data," in *Proceedings of the Int. Conf. on Acoust. Speech and Sig. Proc.*, June 2000.
11. J. Gao and I. Rubin, "Multifractal modeling of counting processes of long-range-dependent network traffic," *Computer Communications* **24**, pp. 1400–1410, 2001.
12. A. Lindenmayer, "Mathematical models for cellular interactions in development II. Simple and branching filaments with two-sided inputs," *Journal of Theoretical Biology* **18**, pp. 300–315, 1968.
13. H. Peitgen, H. Jurgens, and D. Saupe, *Chaos and Fractals: New Frontiers of Science*, Springer-Verlag, 1992.
14. P. Salvador, A. Nogueira, and R. Valadas, "Modeling multifractal traffic with stochastic L-Systems," in *Proceedings of GLOBECOM'2002*, 2002.
15. P. Salvador, A. Nogueira, and R. Valadas, "Modeling multifractal IP traffic: Characterization of packet arrival and sizes with stochastic L-Systems," in *Proceedings of the 10th International Conference on Telecommunication Systems, Modeling and Analysis (ICTSM10)*, **2**, pp. 577–587, 2002.
16. P. Salvador, A. Nogueira, and R. Valadas, "Joint characterization of the packet arrival and packet size processes of multifractal traffic based on stochastic L-Systems," in *Proceedings of the 18th International Teletraffic Congress (ITC18)*, 2003.
17. P. Salvador, A. Nogueira, and R. Valadas, "Scaling behavior analysis of multifractal traffic models based on stochastic L-Systems," tech. rep., University of Aveiro, 2002.