MPLS over WDM Network Design with Packet Level QoS Constraints based on ILP Models

Luís Gouveia, Pedro Patrício
Department of Statistics and Operations Research
Faculty of Sciences, University of Lisbon,
Lisbon, Portugal
lgouveia@fc.ul.pt, pmendes@noe.ubi.pt

Amaro F. de Sousa, Rui Valadas
Institute of Telecommunications
University of Aveiro
Aveiro, Portugal
asou@det.ua.pt, rv@det.ua.pt

***Abstract*— MPLS (Multi-Protocol Label Switching) over WDM (Wavelength Division Multiplexing) networks are gaining significant attention due to the efficiency in resource utilization that can be achieved by jointly considering the two network layers. This paper addresses the design of MPLS over WDM networks, where some of the WDM nodes may not have packet switching capabilities. Given the WDM network topology and the offered traffic matrix, which includes the location of the edge LSRs (Label Switched Routers), we jointly determine the location of the core LSRs (i.e. the core WDM nodes that also need to include packet switching capabilities) and the lightpath routes (which are terminated on the LSRs) that minimize the total network cost. We consider constraints both at the optical and packet layers: an MPLS hop constraint on the maximum number of LSRs traversed by each LSP (Label Switched Path), which guarantees a given packet level QoS, and a WDM path constraint on the maximum length of lightpaths, which accommodates the optical transmission impairments. A novel Integer Linear Programming (ILP) formulation based on an hop-indexed approach, which we call the HOP model, is proposed. A two-phase heuristic, derived from a decomposition of the HOP model in two simpler ILP models that are solved sequentially, is also developed. The computational results show that the heuristic is efficient and produces good quality solutions, as assessed by the lower bounds computed from the HOP model. In some cases, the optimal solution is obtained with the branch-and-bound method.**

Keywords*— *Network design & planning, MPLS over WDM, ILP models

I. INTRODUCTION

Multi-Protocol Label Switching (MPLS) is the technology envisaged for future backbone IP networks to overcome some of the current problems associated with the provision of IP services with Quality of Service (QoS) [1] [2]. In particular, MPLS (i) improves the packet switching performance of routers and (ii) enables traffic engineering by introducing the possibility of source based routing (where the forwarding path from an ingress router to an egress router is not constrained by the paths of other ingress-egress pairs). A router that supports MPLS is known as a Label Switching Router (LSR). MPLS organizes the network in MPLS domains. The forwarding of IP packets from ingress to egress LSRs is done by means of routing paths, called Label Switched Paths (LSPs). In the ingress LSR, incoming IP packets are labeled based on their destination and required QoS and, depending on this classification, are forwarded through the appropriate LSP towards an egress LSR. We consider that edge LSRs (ingress and egress) can also act as intermediate LSRs to LSPs established between other edge LSRs. A packet traveling from ingress to egress undergoes a queuing delay in each LSR it traverses. It has been shown that combining the use of weighted fair queuing scheduling disciplines with leaky bucket traffic shaping at ingress nodes, places a bound on the maximum packet delay which is proportional to the number of LSRs [3]. Thus, QoS delay requirements can be guaranteed by constraining the number of LSRs in each LSP.

At the physical layer, optical transmission and switching technologies are evolving rapidly, offering the prospect for all-optical networks based on Wavelength Division Multiplexing (WDM) and Optical Cross-Connects (OXCs) [4] [5]. In these networks, the optical connectivity between electrical endpoints can be established by all-optical concatenations of WDM channels, called lightpaths. Lightpaths have a limitation on their physical extent due to various transmission impairments (e.g. attenuation, crosstalk, dispersion, nonlinearities).

MPLS over WDM (and IP over WDM) networks are gaining significant attention due to the efficiency in resource utilization that can be achieved by jointly considering the two network layers [6] [7] [8]. These networks are configured by defining lightpaths at the optical layer and LSPs at the packet layer. Lightpaths are routed over the physical network (comprising OXCs connected through optical fibers) and LSPs are routed over the logical topology of lightpaths (the virtual optical network). Besides the OXCs, which provide wavelength switching capabilities, some nodes may also include packet switching capabilities, i.e., some OXCs may have co-located LSRs (these nodes are sometimes called Generalized LSRs [8]). OXCs with co-located LSRs can switch traffic demands between lightpaths, enabling increased resource sharing (i.e. having the traffic demand accommodated in a lower number of lightpaths). In fact, having all OXC sites with co-located LSRs would maximize the resource sharing gains, but this could lead to unnecessarily high network costs.

In this paper we consider a network design problem for MPLS over WDM networks. Given the topology of the WDM layer, the location of the edge nodes and the offered traffic matrix, we determine the core LSR locations (i.e. the core OXCs sites that also need to include packet switching capabilities) and the lightpath routes that minimize the total network cost, subject to constraints at both network layers. We consider (i) an MPLS level QoS constraint given by the maximum number of intermediate LSRs that can be traversed by each LSP and (ii) a WDM path constraint given by the maximum length of each lightpath. In this problem, the network cost includes both the costs of LSR and lightpath placements. The design solution is a tradeoff between the cost of installing LSRs (at OXC sites) and the resource sharing gains that can be obtained from it, which are accounted for by the cost of lightpath placement.

The methodology adopted to solve this problem is based on the derivation of efficient Integer Linear Programming (ILP) models through appropriate reformulation techniques that can be solved with standard ILP algorithms (e.g. branch-and-bound) [9] [10] [11] [12] [13]. Depending on the particular network design problem, this approach has been used by other authors either to determine optimal solutions in realistic times or as a means to derive lower bounds, from the branch-and-bound algorithm, to assess the quality of feasible solutions obtained through heuristics. ILP, and more generally Mixed Integer Linear Programming (MILP), has been widely used in several problems related to the design of optical networks [6] [8] [14] [15].

We model the design problem in an expanded graph that implicitly guarantees the WDM path constraints and use a hop-indexed approach to model the MPLS hop constraints. This is called the HOP model. The hop-indexed approach was previously explored in the context of spanning trees with hop constraints in [12]. We also derive a two-phase heuristic, which is a decomposition of the HOP model in two simpler ILP models that are solved sequentially (to optimality) using the branch-and-bound algorithm. The proposed heuristic was able to solve all considered problem instances in relatively short computing times while the HOP model was able to determine the optimal solutions only in a few cases with large computation times. Moreover, the lower bounds given by the HOP model show that the two-phase heuristic produces good quality solutions and, in some cases, gives the optimal solution.

Network design problems under the context of MPLS network dimensioning were previously considered in [16] and [17]. These problems are solved through heuristics based on Lagrangean relaxation and can be seen as simplified versions of the current network design problem, when considering that all nodes can have co-located LSRs with no additional cost.

This paper is organized as follows: section II defines the network design problem; section III describes the HOP model and discusses different modeling alternatives for the path formulation with hop constraints; in section IV, the two-phase heuristic is derived and explained; and, finally, section V presents the computational results.

II. Characterization of Network Design Problem

Consider (i) an optical network composed by a set of optical cross-connects (OXCs) and a set of fibers connecting the OXCs; (ii) an MPLS overlay packet switching network composed by a set of edge LSRs (which are endpoints for LSPs) and some possible core LSRs (to be determined as part of the design problem), where the connections between LSRs are provided by the optical network; (iii) an offered traffic matrix described in terms of the bandwidth that must be supported between each pair of edge LSRs (i.e., the bandwidth of each LSP). The network design problem consists in determining the number and location of core LSRs and the number and route of lightpaths. Edge LSRs are co-located with OXC nodes; the remaining OXC nodes are candidate locations for core LSRs. The objective of the network design problem is to obtain the least cost solution and, since the WDM network topology and the edge LSRs are given, the cost of each solution varies with the number and location of the required core LSRs and the number and length of the required lightpaths. We consider a cost value for each core

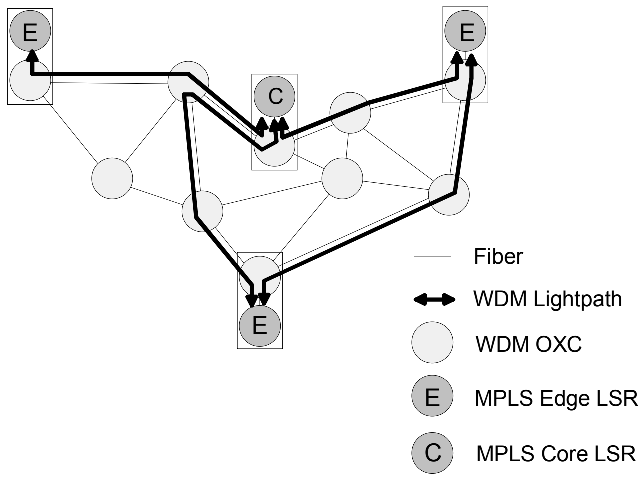

Figure 1. Virtual optical network over the WDM network.

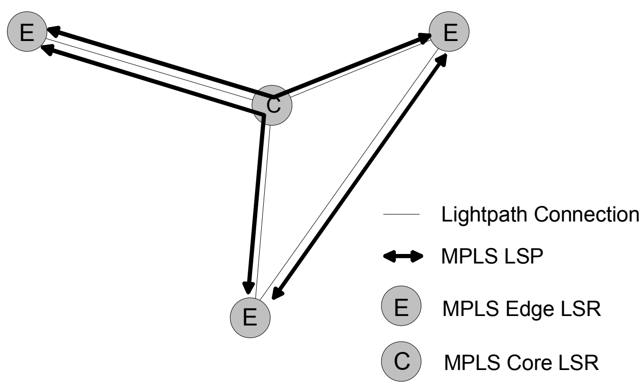

Figure 2. MPLS network over the virtual optical network.

LSR in each candidate location and a cost value per length unit for each lightpath in the solution.

Figures 1 and 2 illustrate a feasible solution with a single core LSR of the network design problem. Figure 1 represents the virtual optical network of lightpaths over the WDM network, LSRs are end nodes for lightpaths and OXCs are optical switching nodes for lightpaths. Figure 2 represents the corresponding MPLS network layer where lightpaths are seen as point-to-point connections between LSRs, edge LSRs are end nodes for LSPs and core LSRs are packet switching nodes for LSPs.

A network design configuration is a feasible solution if it satisfies the following three constraints:

Network loading constraints: the total capacity of the lightpaths between two LSRs must be not less than the sum of the bandwidth capacities of the LSPs that cross them;

MPLS hop constraints: an LSP between a given pair of edge LSRs must have a number of intermediate LSRs (either core LSRs or other edge LSRs) not greater than a predefined number;

WDM path constraints: the route of a lightpath between two LSRs (either core or edge) must be not greater than a given maximum length value.

The network design problem is the determination of the least cost network solution among the set of all feasible solutions, i.e., the solutions that comply with the above three constraints.

III. HOP MODEL FOR THE NETWORK DESIGN PROBLEM

Let the WDM network layer be modeled by the graph $N = (X,E)$ where the node set X represents the OXC locations and the edge set E represents the pairs of OXCs connected by optical fibers. Some of the nodes belonging to X have co-located edge LSRs which are represented by node set ER ($ER \subset X$). The set $X \backslash ER$ denotes the set of candidate core LSR locations. At the MPLS network layer, an LSP between each pair of nodes belonging to ER must be supported with a given bandwidth (in this work, we consider symmetrical LSPs). Therefore, we associate to each pair of edge LSRs an LSP k with traffic demand b_k that must be supported between the corresponding source edge LSR, $s_k \in ER$, and target edge LSR, $t_k \in ER$. K denotes the set of all LSPs. The WDM path constraints impose that the length of a lightpath connecting any two LSRs cannot be higher than a maximum value, denoted by H1. The MPLS hop constraints impose a maximum number of H2 intermediate LSRs (either edge or core) in the path of any LSP k. The cost value c_i denotes the cost of placing a core LSR in operation at node $i \in X \backslash ER$. The cost value $c_{\{ij\}}$ denotes the cost of placing a lightpath in operation between two LSRs located in nodes i and j (we consider this cost value to be proportional to the lightpath length).

Given that the only constraint involving the lightpaths is the maximum length and that the cost of a lightpath is proportional to its length, it is straightforward to observe that, in the optimal solution of the problem, a lightpath between any two nodes is always routed along the shortest path on graph N. This observation permits us to model the network design problem in an expanded graph $N' = (X,E')$ with the same node set as N and where edge $e = \{i,j\}$ is included in E' if and only if the shortest path in N between i and j is not greater than H1. Figure 3 gives an example of an Euclidean graph N with 10 nodes and 18 edges (where white nodes represent OXC locations and dark nodes represent OXC locations with co-located edge LSRs) and the resulting expanded graph N' where the edge set has 4 additional edges represented by thicker lines (both graphs are defined in a square of size 1.5 units of H1).

The edges of the expanded graph N' represent pairs of nodes that can be lightpath endpoints, i.e., such that LSRs can be located at these nodes and be connected through lightpaths. The advantage of modeling the problem in the expanded graph N' is that the WDM path constraints are implicitly guaranteed

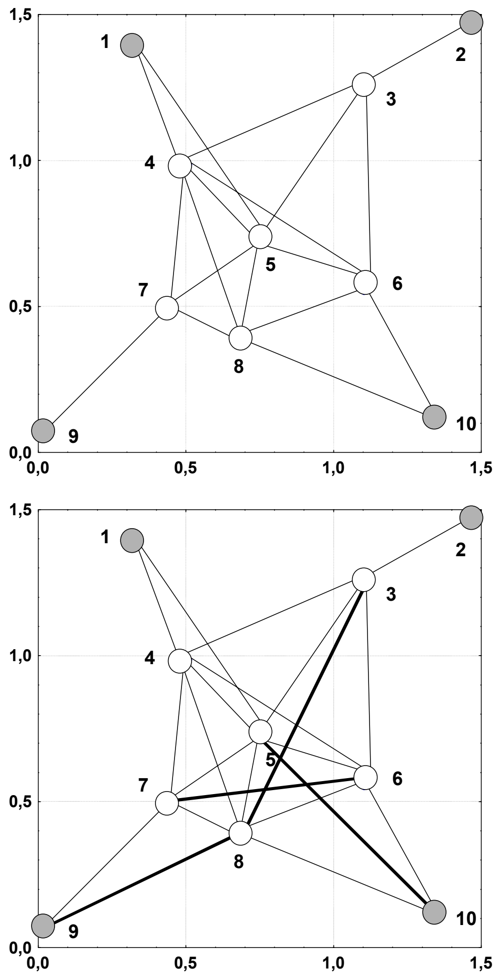

Figure 3. Example of an Euclidean graph N (above) and the corresponding expanded graph N' (below).

in this new graph and do not need to be explicitly modeled in the Integer Linear Programming (ILP) model of the problem.

We present next a *generic* model for the design problem defined in graph N'. The model involves the following set of variables, which completely define a solution for the network design problem:

N_i – binary variable that equals 1 if a core LSR is to be put in operation in node $i \in X \backslash ER$; 0 otherwise.

y_{ij}^k – binary variable that equals 1 if edge $\{i,j\} \in N'$ is traversed by LSP $k \in K$ in the direction from i to j; 0 otherwise.

$u_{\{ij\}}$ – integer variable specifying the number of lightpaths installed in edge $\{i,j\} \in N'$ (i.e., the number of lightpaths connecting an LSR placed in node $i \in X$ and an LSR placed in node $j \in X \setminus \{i\}$).

Let $V(j)$ denote the set of neighbor nodes of node j in graph N', i.e., $V(j)$ is the set of nodes $i \in X$ such that edge $\{i,j\}$ belongs to edge set E'. The *generic* model for the design problem, defined in graph N', is given by:

Generic Model:

$$Min \sum_{i \in X \setminus ER} c_i N_i + \sum_{\{i,j\} \in E'} c_{\{ij\}} u_{\{ij\}} \qquad (1.1)$$

subject to:

$$\left\{ y_{ij}^k : y_{ij}^k = 1 \right\} \text{ define a path for LSP } k \in K \text{ from } s_k \text{ to } t_k \text{ traversing at most H2 nodes} \qquad (1.2)$$

$$\sum_{i \in V(j)} y_{ij}^k \le N_j \,, \; j \in X \setminus ER, \; k \in K \qquad (1.3a)$$

$$\sum_{i \in V(j)} y_{ij}^k \le 1 \,, \; j \in ER, \; k \in K \qquad (1.3b)$$

$$\sum_{k \in K} b_k \left(y_{ij}^k + y_{ji}^k \right) \le \alpha \cdot u_{\{ij\}} \,, \; \{i,j\} \in E' \qquad (1.4)$$

$$y_{ij}^k + y_{ji}^k \le u_{\{ij\}} \,, \; \{i,j\} \in E', k \in K \qquad (1.5)$$

$$N_i \in \{0,1\}, \; i \in X \setminus ER \qquad (1.6a)$$

$$u_{\{ij\}} \ge 0 \text{ and integer}, \; \{i,j\} \in E' \qquad (1.6b)$$

$$y_{ij}^k, y_{ji}^k \in \{0,1\}, \; \{i,j\} \in E', k \in K \qquad (1.6c)$$

Constraints (1.2) are given in a generic way and different sets of linear inequalities for describing them are discussed later. Constraints (1.3a) guarantee that if the path of at least one LSR $k \in K$ includes a given node j that has not an edge LSR, then a core LSR must be put in operation at that node. These constraints together with constraints (1.3b) guarantee that the LSP paths do not repeat nodes, i.e., they do not contain cycles. Constraints (1.4) are the network loading constraints and guarantee that there are enough lightpaths between each LSR pair (α is the bandwidth of a single lightpath) to support the sum of the bandwidths of all LSPs that use these lightpaths. Constraints (1.5) are additional constraints (not necessary for defining the network design problem) that help significantly to solve the model by available ILP solvers. These constraints simply state that we must install a lightpath in an edge $\{i,j\}$ if the edge is traversed by at least one LSP. Constraints (1.6a/b/c) define the variables of the model.

Figure 4 shows an optimal solution for the example introduced in Figure 3 with H2 = 3 (black nodes represent the locations of core LSRs selected by the model and edges represent the lightpaths connecting LSRs). Since the model is defined on the expanded graph N', the solution edges connect directly nodes with co-located LSRs (Figure 4 above). To compute the network design solution of the original problem, each edge must be replaced by the shortest path connecting its end nodes on the original graph (Figure 3 above). For example, in the solution shown in Figure 4 (above), the edge

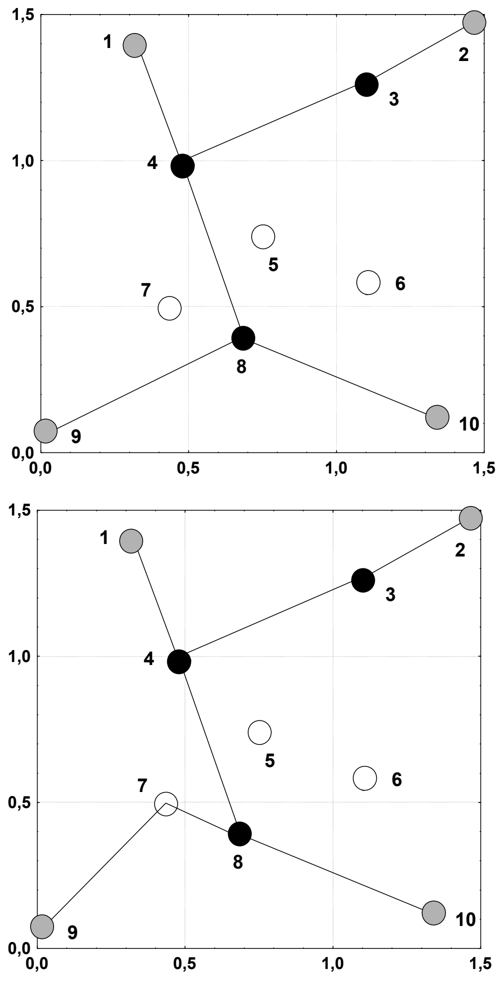

Figure 4. Solution of example from Figure 3 in the expanded graph N' (above) and in the original graph N (below).

solution connecting node 9 to node 8 means that a number of lightpaths (given by the solution value of variable $u_{\{ij\}}$, $i = 8$ and $j = 9$) must be set-up between edge LSR located in node 9 and a core LSR that must be placed in operation in node 8; since this edge is not in the original graph N (Figure 3 above), these lightpaths must be routed through the shortest path in the original graph, which is via the OXC of node 7 as represented in Figure 4 (below).

One way of modeling generic constraints (1.2) is the *constrained path* model [9] [12] that uses the flow variables y_{ij}^k as already defined in the generic model:

$$\sum_{j \in V(i)} y_{ij}^k - \sum_{j \in V(i)} y_{ji}^k = \begin{cases} 1, & i = s_k \\ 0, & i \neq s_k, t_k \\ -1, & i = t_k \end{cases}, \; k \in K \quad \text{(A.1)}$$

$$\sum_{(i,j):\{i,j\} \in E'} y_{ij}^k \leq \text{H2} + 1, \; k \in K \quad \text{(A.2)}$$

For each LSP $k \in K$, the flow conservation constraints (A.1) guarantee that the variables y_{ij}^k include a path from its source node s_k to its target node t_k and the cardinality constraints (A.2) guarantee that the path crosses at most H2+1 edges which is equivalent to say the number of intermediate nodes is at most H2.

Another way of modeling generic constraints (1.2) is the *feasible path* model [11] [13], that uses the flow variables y_{ij}^k defined in the generic model together with binary variables f_p^k that indicate whether the p^{th} feasible path for LSP k (from source node s_k to target node t_k) is included in the solution (we assume that the feasible paths are ordered for each LSP $k \in K$):

$$\sum_{p \in P_k} f_p^k = 1, \qquad k \in K \quad \text{(B.1)}$$

$$\sum_{p \in P_{ij}^k} f_p^k = y_{ij}^k, \qquad k \in K \; ; \; \{i,j\} \in E' \quad \text{(B.2)}$$

$$f_p^k \in \{0,1\}, \qquad k \in K \; ; \; p \in P_k \quad \text{(B.3)}$$

In the *feasible path* model, P_k represents the set of feasible paths for LSP k (that is, the paths with at most H2 intermediate nodes in graph N' from the source node s_k to the target node t_k) and P_{ij}^k represent the set of feasible paths of LSP k that use edge $\{i,j\} \in E'$ in the direction from i to j. Constraints (B.1) guarantee that the solution contains one feasible path for each LSP k and constraints (B.2) relate the path variables with the flow variables of the generic model: the flow variable y_{ij}^k is 1 if the selected feasible path of LSP k uses edge $\{i,j\}$ in the direction from i to j.

In this work, we adopted a more recent approach called *hop-indexed* model that was previously explored in the context of spanning trees with hop constraints [12] [18]. This model is a formulation that besides the flow variables defined in the generic model, also uses binary variables $z_{ij,p}^k$ indicating whether LSP $k \in K$ traverses edge $\{i,j\}$ in the direction from i to j and $\{i,j\}$ *is the* p^{th} *edge in the path* from the source node s_k to the target node t_k:

$$\sum_{j \in V(s_k)} z_{s_k j,1}^k = 1, \quad k \in K \quad \text{(C.1)}$$

$$\sum_{j \in V(i)} z_{ij,p}^k - \sum_{j \in V(i)} z_{ji,p-1}^k = 0, \; i \neq s_k, t_k; \; k \in K; \; p = 2,\ldots,\text{H2}+1 \quad \text{(C.2)}$$

$$z_{t_k t_k,\text{H2}+1}^k + \sum_{j \in V(t_k)} z_{j t_k,\text{H2}+1}^k = 1, \quad k \in K \quad \text{(C.3a)}$$

$$z_{t_k t_k,p}^k = z_{t_k t_k,p-1}^k + \sum_{j \in V(t_k)} z_{j t_k,p-1}^k, \; k \in K; \; p = 2,\ldots,\text{H2}+1 \quad \text{(C.3b)}$$

$$y_{ij}^k = \sum_{p=1}^{\text{H2}+1} z_{ij,p}^k, \; \{i,j\} \in E', \; k \in K \quad \text{(C.4)}$$

$$z_{s_k j,1}^k \in \{0,1\}, \; \{s_k, j\} \in E'; \; k \in K \quad \text{(C.5a)}$$

$$z_{ij,p}^k \in \{0,1\}, \; \{i,j\} \in E' \wedge i \neq s_k; \; k \in K; \; p = 2,\ldots,\text{H2}+1 \quad \text{(C.5b)}$$

$$z_{t_k t_k,p}^k \in \{0,1\}, \; k \in K; \; p = 2,\ldots,\text{H2}+1 \quad \text{(C.5c)}$$

This model contains "loop" variables $z_{t_k t_k,p}^k$, $p = 2, \ldots,$ H2+1, in the target node t_k to model the cases when the path from the source node s_k to the target node t_k contains fewer than H2+1 arcs (that is, $z_{j t_k,p}^k = 1$ for some $p <$ H2+1). For each LSP $k \in K$, the *hop-indexed* model constraints guarantee that:

Constraints (C.1): one of the edges coming out of the source node s_k must be the 1^{st} edge in its path;

Constraints (C.2): for all nodes except the source and target nodes, an edge that goes into the node as the $(p\text{-}1)^{th}$ edge in the path forces an edge coming out of that node to be also in the path as the p^{th} edge;

Constraints (C.3a/b): the path includes an edge going into the target node t_k in a position less or equal to H2+1: constraints (C.3a) guarantee that the path ends through an edge in position H2+1 (either coming from a neighbor node or from a "loop" variable) and constraints (C.3b) guarantee that the loop variable associated with the p^{th} position must be 1 either if there is an edge going into the target node t_k in the $(p\text{-}1)^{th}$ position or if the loop variable associated with the $(p\text{-}1)^{th}$ position is already 1.

Constraints (C.4): relate the hop-index variables with the flow variables of the generic model: the flow variable y_{ij}^k is 1 if for a given p =1,…, H2+1, the hop-indexed variables $z_{ij,p}^k$ equals 1.

The *hop-indexed* model is equivalent to an unconstrained path model in a layered acyclic network. For better understanding, consider a fully connected network with 5 nodes and an LSP $k \in K$ with origin $s_k = 1$ and destination $t_k = 5$. If we consider H2 = 3, figure 5 represents the corresponding layered graph N_k that contains H2 copies of each node $i \notin \{s_k, t_k\}$, named (i,p), and H2+1 copies of node t_k. A node (i,p) is visited in the unconstrained path of the layered network if and only if node i is visited in position p in the hop constrained path in the original network. Traversing an arc $((i,p),(j,p+1))$, with $i \neq j$ and p = 0,...,H2, in the layered graph corresponds to traversing the arc (i,j) in position p+1 in the path of the original network. Note that the layered network contains arcs of the form (t_k,p), $(t_k,p+1)$, with p=1,...,H2. These arcs correspond to the previously mentioned loop variables. In figure 5, a 2-hop path in the original network that includes arcs (1,3) and (3,5) is represented by the path shown in bold. The variables with value equal to one representing this path in the *hop-indexed* model are $z_{13,1}^k, z_{35,2}^k, z_{55,3}^k\ z_{55,4}^k$.

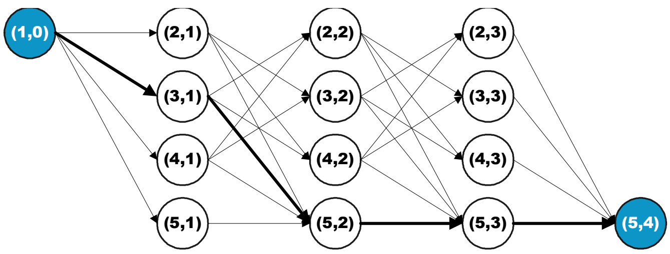

Figure 5. The layered network N_k.

Note that a feasible path in the layered network can include any two nodes (i,p) and (i,q) for $q>p+1$ which means that nodes can be repeated in a path in the original network. However, the hop-indexed constraints together with constraints (1.3a/b) in the complete model guarantee that these repetitions are not allowed.

It can be shown that the generic model with either the *feasible path* model or the *hop-indexed* model for modeling constraints (1.2) has the same Linear Programming (LP) relaxation value (the LP relaxation of an ILP model is obtained from the original model by considering the integer variables as real variables between 0 and their maximum integer value). The proof of this equivalence result is given in [19] for a slightly different problem. The same result holds in the context of more general network design problems with hop constraints and the problem studied in this paper fits into this general class. There is a clear advantage of using the *hop-indexed* model instead of the *feasible path* model for modeling constraints (1.2): the former has a compact number of variables and constraints and thus, can be solved by using available packages for solving ILP models; the later has an exponential number of variables (one for each feasible path) that requires either a type of implicit generation scheme that must be embedded with the solver or some “a priori” heuristic selection of path variables to be included in the model (note that this second option may loose optimality).

Concerning the *constrained path* model, the *hop-indexed* model is much more complex since it contains far more variables and constraints. However, some interesting properties of the hop-index model presented and discussed in [12] indicate that modeling constraints (1.2) with the more complex *hop-indexed* model instead of the simpler *constrained path* model makes the whole model easier to solve (or, in other words, reduces the computing time) when using a standard branch-and-bound algorithm. The main reason for this fact is that the LP relaxation value of the *hop-indexed* model is much closer to the optimum integer value which, in turn, gives a better first solution for the branch-and-bound to start with. Our computational experience, using the *hop-indexed* model, on this network design problem has confirmed these expectations.

For these reasons, the remainder of the paper only addresses the generic model with (1.2) modeled by the *hop-indexed* model. For completeness reasons, we present below the generic model with (1.2) substituted by the constraints defining the *hop-indexed* model and designate this as the HOP model. Note that the equalities given by constraints (C.4) can be used to redefine constraints (1.3a/b), (1.4) and (1.5) in order to eliminate the original flow variables y_{ij}^k. For better understanding, we maintain in the HOP model the numbering of constraints as given in the *generic* model and in the *hop-indexed* part of the model (where the letter C is replaced by 1.2).

HOP Model:

$$Min \sum_{i \in X \setminus ER} c_i N_i + \sum_{\{i,j\} \in E'} c_{\{ij\}} u_{\{ij\}} \qquad (1.1)$$

subject to:

$$\sum_{j \in V(s_k)} z_{s_k j,1}^k = 1 \ , \quad k \in K \qquad (1.2.1)$$

$$\sum_{j \in V(i)} z_{ij,p}^k - \sum_{j \in V(i)} z_{ji,p-1}^k = 0 \ , \ i \neq s_k, t_k ; \ k \in K ; \ p = 2,..., \text{H2}+1 \qquad (1.2.2)$$

$$z_{t_k t_k, \text{H2}+1}^k + \sum_{j \in V(t_k)} z_{j t_k, \text{H2}+1}^k = 1 \ , \quad k \in K \qquad (1.2.3a)$$

$$z^k_{t_k t_k,p} = z^k_{t_k t_k,p-1} + \sum_{j \in V(t_k)} z^k_{j t_k,p-1} \quad ,k \in K;\ p = 2,...,\text{H2}+1 \quad (1.2.3b)$$

$$\sum_{i \in V(j)} \sum_{p=1}^{\text{H2}+1} z^k_{ij,p} \le N_j \ ,\ j \in X \setminus ER,\ k \in K \quad (1.3a)$$

$$\sum_{i \in V(j)} \sum_{p=1}^{\text{H2}+1} z^k_{ij,p} \le 1 \ ,\ j \in ER,\ k \in K \quad (1.3b)$$

$$\sum_{k \in K} b_k \left(\sum_{p=1}^{\text{H2}+1} z^k_{ij,p} + \sum_{p=1}^{\text{H2}+1} z^k_{ji,p} \right) \le \alpha \cdot u_{\{ij\}},\ \{i,j\} \in E' \quad (1.4)$$

$$\sum_{p=1}^{\text{H2}+1} z^k_{ij,p} + \sum_{p=1}^{\text{H2}+1} z^k_{ji,p} \le u_{\{i,j\}} \ ,\ \{i,j\} \in E',\ k \in K \quad (1.5)$$

$$N_i \in \{0,1\},\ i \in X \setminus ER \quad (1.6a)$$

$$u_{\{ij\}} \ge 0 \text{ and integer},\ \{i,j\} \in E' \quad (1.6b)$$

$$z^k_{s_k j,1} \in \{0,1\},\ \{s_k, j\} \in E';\ k \in K \quad (1.2.5a)$$

$$z^k_{ij,p} \in \{0,1\},\ \{i,j\} \in E' \wedge i \ne s_k;\ k \in K;\ \ p = 2,...,\text{H2}+1 \quad (1.2.5b)$$

$$z^k_{t_k t_k,p} \in \{0,1\},\ k \in K;\ p = 2,...,\text{H2}+1 \quad (1.2.5c)$$

The network design problem defined by this ILP model can be easily extended to cope with multi-service MPLS networks. In the general case, we associate to each pair of edge LSRs a set of LSPs (one for each service) instead of a single LSP. Furthermore, for each one of these LSPs we may associate a different value of parameter H2 in cases with services with different QoS packet delay requirements.

IV. A TWO-PHASE HEURISTIC

Based on the HOP model, we have developed a two-phase heuristic that can be used to determine feasible solutions for the network design problem. The reasons for this approach are as follows: (i) the cost value of the solutions obtained with the heuristic can be given to the branch-and-bound algorithm as an upper bound, to help solving the HOP model (this can reduce significantly the computational time); (ii) the heuristic can obtain solutions for problem instances that could not be solved to optimality by the HOP model, due to exaggerated computational times or memory failure.

The two-phase heuristic is based on the decomposition of the HOP model in two simpler ILP models that are solved one after the other. The first phase of the heuristic ignores the part of the problem that relates to lightpaths. It focus on finding locations for the core LSRs in order to guarantee that, for each pair of edge LSRs, there is at least one path satisfying the MPLS hop constraints. The objective is to minimize the part of the original objective function associated with the costs of core LSRs. The model is again an ILP which is a restricted version of the HOP model where the lightpath costs are ignored in the objective function (1.1) and the constraints involving variables $u_{\{ij\}}$ (constraints (1.4), (1.5) and (1.6b)) are eliminated.

Phase 1 model:

$$Min \sum_{i \in X \setminus ER} c_i N_i \quad (2.1)$$

subject to:

$$\sum_{j \in V(s_k)} z^k_{s_k j,1} = 1\ ,\ \ k \in K \quad (2.2.1)$$

$$\sum_{j \in V(i)} z^k_{ij,p} - \sum_{j \in V(i)} z^k_{ji,p-1} = 0,\ i \ne s_k, t_k;\ k \in K;\ p = 2,...,\text{H2}+1 \quad (2.2.2)$$

$$z^k_{t_k t_k,\text{H2}+1} + \sum_{j \in V(t_k)} z^k_{j t_k,\text{H2}+1} = 1\ ,\ \ k \in K \quad (2.2.3a)$$

$$z^k_{t_k t_k,p} = z^k_{t_k t_k,p-1} + \sum_{j \in V(t_k)} z^k_{j t_k,p-1} \quad ,k \in K;\ p = 2,...,\text{H2}+1 \quad (2.2.3b)$$

$$\sum_{i \in V(j)} \sum_{p=1}^{\text{H2}+1} z^k_{ij,p} \le N_j \ ,\ j \in X \setminus ER,\ k \in K \quad (2.3a)$$

$$\sum_{i \in V(j)} \sum_{p=1}^{\text{H2}+1} z^k_{ij,p} \le 1 \ ,\ j \in ER,\ k \in K \quad (2.3b)$$

$$N_i \in \{0,1\},\ i \in X \setminus ER \quad (2.6a)$$

$$z^k_{s_k j,1} \in \{0,1\},\ \{s_k, j\} \in E';\ k \in K \quad (2.2.5a)$$

$$z^k_{ij,p} \in \{0,1\},\ \{i,j\} \in E' \wedge i \ne s_k;\ k \in K;\ \ p = 2,...,\text{H2}+1 \quad (2.2.5b)$$

$$z^k_{t_k t_k,p} \in \{0,1\},\ k \in K;\ p = 2,...,\text{H2}+1 \quad (2.2.5c)$$

The second phase of the heuristic considers the locations of core LSRs given by the optimal solution of the previous phase (defined by the solution of variables N_i) as input parameters. This phase is concerned with the determination of the required lightpaths (how many and between which LSRs). The output of phase 1 guarantees that a feasible solution can be obtained. We consider a restricted version of *N'*, the network *N''* = (*ER*∪*CR*,*E''*), where *CR* denotes the set of core LSR locations obtained in the previous phase. The set of edges *E''* is a subset of *E'* corresponding to the edges that have both endpoints in *ER*∪*CR*. Considering again the example of Figure 3, we show in Figure 6 the network *N''* that has been generated assuming that the solution of phase 1 is a set of three core LSRs in nodes 3, 5 and 8 (this is different from Figure 4, which represents a solution obtained with the HOP

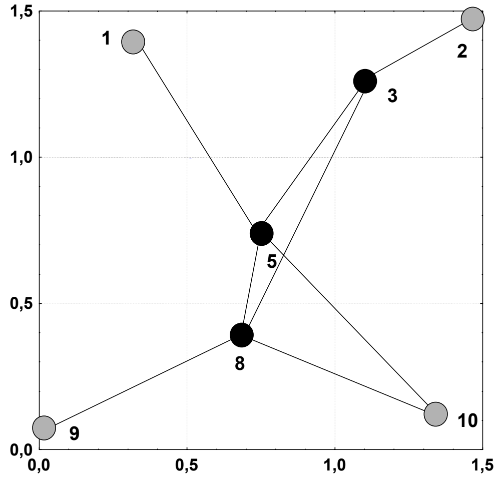

Figure 6. Graph *N''* corresponding to the example of Figure 3.

model). Note that since phase 1 is based only on the cost of core LSRs, it can produce a set of locations that may not allow the optimal solution of the whole problem to be obtained in phase 2. Therefore, the described two-phase procedure does not guarantee the determination of the optimal network design solution.

The second phase is also modeled by an ILP and the problem is defined in graph *N''*. The set *V*(*j*) represents, now, the set of neighbors of node *j* on graph *N''*. This second phase model is again a restricted version of the HOP model where the core LSR costs are ignored in the objective function (1.1) and the constraints involving variables N_i (constraints (1.3a) and (1.6a)) are eliminated.

Phase 2 model:

$$Min \sum_{\{i,j\}\in E''} c_{\{ij\}} u_{\{ij\}} \quad (3.1)$$

subject to:

$$\sum_{j\in V(s_k)} z^k_{s_k j,1} = 1\,, \quad k\in K \quad (3.2.1)$$

$$\sum_{j\in V(i)} z^k_{ij,p} - \sum_{j\in V(i)} z^k_{ji,p-1} = 0\,, \; i\neq s_k, t_k; \; k\in K; \; p=2,...,\mathrm{H2}+1 \quad (3.2.2)$$

$$z^k_{t_k t_k,\mathrm{H2}+1} + \sum_{j\in V(t_k)} z^k_{jt_k,\mathrm{H2}+1} = 1\,, \; k\in K \quad (3.2.3a)$$

$$z^k_{t_k t_k,p} = z^k_{t_k t_k,p-1} + \sum_{j\in V(t_k)} z^k_{jt_k,p-1} \;, k\in K\,; \; p=2,...,\mathrm{H2}+1 \quad (3.2.3b)$$

$$\sum_{i\in V(j)} \sum_{p=1}^{\mathrm{H2}+1} z^k_{ij,p} \leq 1\,, \; j\in ER\cup CR, \; k\in K \quad (3.3b)$$

$$\sum_{k\in K} b_k \left(\sum_{p=1}^{\mathrm{H2}+1} z^k_{ij,p} + \sum_{p=1}^{\mathrm{H2}+1} z^k_{ji,p} \right) \leq \alpha\cdot u_{\{ij\}}, \; \{i,j\}\in E'' \quad (3.4)$$

$$\sum_{p=1}^{\mathrm{H2}+1} z^k_{ij,p} + \sum_{p=1}^{\mathrm{H2}+1} z^k_{ji,p} \leq u_{\{i,j\}} \;, \{i,j\}\in E'', k\in K \quad (3.5)$$

$$u_{\{ij\}} \geq 0 \text{ and integer}, \{i,j\}\in E'' \quad (3.6b)$$

$$z^k_{s_k j,1} \in \{0,1\}, \{s_k, j\}\in E'; \; k\in K \quad (3.2.5a)$$

$$z^k_{ij,p} \in \{0,1\}, \; \{i,j\}\in E' \wedge i\neq s_k\,; k\in K; \; p=2,...,\mathrm{H2}+1 \quad (3.2.5b)$$

$$z^k_{t_k t_k,p} \in \{0,1\}, \; k\in K; \; p=2,...,\mathrm{H2}+1 \quad (3.2.5c)$$

Note that constraints (3.3b) in this second phase model, equivalent to constraints (1.3b) of the HOP model, are extended to all nodes of *N''* since in this graph there are LSRs in all nodes.

V. COMPUTATIONAL RESULTS

To analyze the performance of the HOP model and of the two-phase heuristic, we have generated 4 Euclidean networks: 2 networks with 25 OXC nodes (networks N25a and N25b) and 2 networks with 50 OXC nodes (networks N50a and N50b).

The OXC nodes were randomly located in a square grid of dimension 2 by 2 for the 25 node instances and 2.5 by 2.5 for the 50 node instances (the dimensions are given in number of H1 units, where H1 is the maximum lightpath length). The 25 node instances were generated with 50 edges and the 50 node instances with 100 edges. To generate the optical fiber links of each network we considered, first, the complete graph generated by all OXC nodes and assigned costs to each link equal to the integer part of the Euclidean distance between its end nodes. Then, we determined the minimum cost spanning tree in this graph. The edges of this spanning tree are included in the WDM network together with the cheapest remaining 50-(25-1) and 100-(50-1) edges, respectively for the 25 and 50 node instances.

At the MPLS network layer, we have considered 12 edge LSRs for instances with 25 nodes (generating a traffic matrix with 66 origin-destination pairs), and 15 edge LSRs for instances with 50 nodes (generating a traffic matrix with 105 origin-destination pairs). The majority of edge LSRs were located at the network fringes, to make sure that the WDM path and MPLS hop constraints have some effect on the feasible solutions (if edge LSRs were all located near each other, direct lightpaths between them would be feasible making the problem a lot easier to solve). Thus, the following rules were used: in network N25a, the 12 edge LSRs were located at the most distant nodes from the Euclidean center of the network; in network N25b, 10 edge LSRs were located at the most distant nodes from the Euclidean center and 2 edge LSRs were located closer to the Euclidean center; in network

N50a, 14 edge LSRs were located at the most distant nodes from the Euclidean center and 1 edge LSR was located closer to the Euclidean center; in network N50b, 12 edge LSRs were located at the most distant nodes from the Euclidean center and 3 edge LSRs were located closer to the Euclidean center. Figure 7 presents the resulting optical topology and edge LSR locations for the networks N25b and N50b.

In all problem instances, we assumed that the cost of putting a core LSR in operation is $c_i = 100$, for all $i \in X\backslash ER$; the cost of a lightpath between nodes i and j is given by $c_{\{ij\}} = 20 \times$ (*length of the shortest path between i and j*) (in H1 units); the capacity of lightpaths is $\alpha = 1$; the bandwidth demands b_k, of all LSP $k \in K$, are randomly generated with an uniform distribution in the interval]0, 0.1]. For all problem instances, we considered two values for the parameter H2.

The edge set E' of the expanded network N' is computed

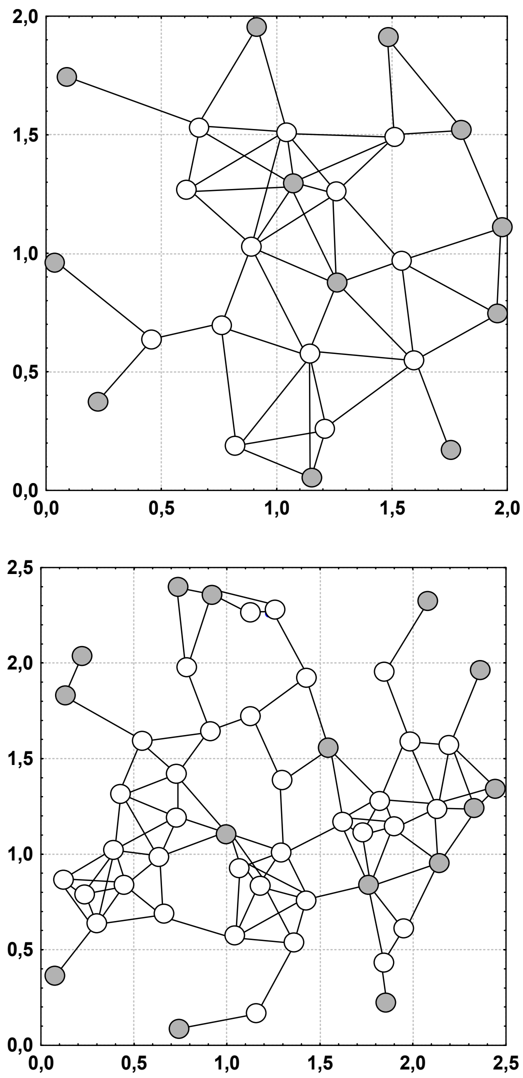

Figure 7. Network N25b (above) and network N50b (below).

apriori, which resulted in expanded graphs of 109 edges (network N25a), 125 edges (network N25b), 367 edges (network N50a) and 388 edges (network N50b).

As stated before, both the HOP model and the phase 2 model of the heuristic contain additional constraints (constraints (1.5) and (3.5)) that increase significantly the LP relaxation value of the models and therefore give a better first solution for the branch-and-bound to start with. Unfortunately, these sets contain a large number of constraints and our experience indicates that including all of them in the models would lead to an excessive computational effort. Thus, instead of adding all inequalities to the models from the beginning, we have decided to consider only a selected sub-set C. This set C is computed as follows. We start by solving the LP relaxation of the HOP model without constraints (1.5), or their equivalent in phase 2 of the heuristic (the LP relaxation of an ILP model is much easier to solve than the original model [20]). The optimal solution of the LP relaxation is examined to see whether it violates some of the constraints (1.5). The violated constraints are added to the model (as noted before, these constraints help to improve the LP relaxation but do not change the value of the optimal integer solution). We solve the LP relaxation of the new model and proceed as before, that is, we examine whether some of the remaining inequalities in (1.5) are violated by the new LP solution. If there are some violated inequalities, the process continues. If none is violated by the current optimal LP solution, the procedure stops and the set C is then defined by all the inequalities added to the model in all iterations. For example, the problem instance of network N25a with H2 = 3 was solved to optimality in 588 minutes when all constraints (1.5) were included in the model but it took only 66 minutes (Table II) when the sub-set C was considered. In this case, the above method for generating constraints was able to reduce the 7194 constraints of type (1.5) included in the original model to only 422 constraints (about 6% of the total). All the computational results presented in this section make use of this constraint generation procedure.

All results were obtained through the use of CPLEX 7.0 (a standard solver package that includes the branch-and-bound algorithm for solving ILP models) running on a PC platform with a Pentium III processor at 450Mhz of clock rate and with 256MB of RAM.

Table I presents the solution values (SV) and the CPU times (CPU) obtained in phase 1 and phase 2 of the proposed heuristic, and the complete solution value (CSV). The CPU times show that the phase 1 model is easy to solve. The phase 2 model is much harder to solve which suggests that additional research is necessary to improve its efficiency. Nevertheless, the smaller cases were solved in less than half an hour and the most difficult case (N50a with H2 = 4) was solved in around 6 hours of CPU time which is still a realistic time for a network design task. We note that there are no feasible solutions for H2 values lower than the ones presented in Table I, since for these H2 values there is no network configuration that can cope simultaneously with the WDM path constraints and MPLS hop constraints.

TABLE I. COMPUTATIONAL RESULTS OF THE HEURISTIC

	H2	Two-Phase Heuristic				CSV
		Phase 1		*Phase 2*		
		SV	*CPU*	*SV*	*CPU*	
N25a	3	300	4 sec	194	2 min	494
	4	300	5 sec	185	21 min	485
N25b	2	200	1 sec	215	1 min	415
	3	200	3 sec	199	13 min	399
N50a	3	400	19 sec	340	4 min	740
	4	300	83 sec	360	347 min	660
N50b	3	600	18 sec	326	240 min	926
	4	400	48 sec	314	107 min	714

TABLE II. COMPUTATIONAL RESULTS OF HOP MODEL

	H2	HOP Model		Gap
		SV	*CPU*	
N25a	3	481	66 min	2.70%
	4	454.24 *	2 days	6.77%
N25b	2	415	2 min	0.00%
	3	383.5 *	2 days	4.04%
N50a	3	740	1438 min	0.00%
	4	599.8 *	2 days	10.03%
N50b	3	906.75 *	2 days	2.12%
	4	683.13 *	2 days	4.52%

Table II presents the results obtained by the HOP model (SV and CPU have the same meaning as in Table I). In this case, we imposed a limit of 2 days on the maximum CPU time and gave an initial upper bound to the branch-and-bound algorithm given by the solution values of the two-phase heuristic (presented in Table I). The results with the symbol * refer to the cases where the HOP model did not achieve the optimal solution (in fact, none of these cases achieved any feasible solution below the upper bound given by the heuristic). In these cases, the SV values shown in Table II are the best lower bound values obtained by the branch-and-bound algorithm. Either the value of the optimal solution or the lower bound value (obtained through the HOP model) can be used to assess the quality of the heuristic solution. In Table II, the "Gap" column gives, in percentage, the difference between the cost value of the solution obtained with the heuristics and the SV value obtained with the HOP model.

The results of Table II show that the HOP model was able to obtain the optimal solution only in three cases. Nevertheless, in two of these cases, the heuristic achieved the optimal solution and, in the other case, a feasible solution which is only 2.7% worse than the optimum value was obtained in around 2 minutes while the optimum solution took 66 minutes. In the cases where the HOP model did not yield the optimal solution, the reported lower bounds show that the solutions of the heuristic are at most (in the worst case) 10% above the optimal value. Note, however, that in these cases the quality of the solutions may be better than suggested by the gap values, since we do not know whether the optimal value is near the lower bound or near the value of the heuristic solution.

The main advantage of the proposed decomposition approach is that it separates the original problem into (i) a hop-indexed facility allocation problem (which is easy to solve) and (ii) a standard multi-commodity network loading problem defined in a much smaller graph and, therefore, much easier to solve than the original one.

Note also that the two-phase heuristic favors solutions that minimize the costs of core LSRs. This is a good approach since, in general, electrical switching involves higher costs than optical switching and, therefore, the overall cost of a design solution is more dependent on the costs of LSRs than the costs of lightpaths.

The CPU times of Table I and II show that the adopted models are more efficient (have on average smaller computing times) for smaller values of H2. This result was expected since the models become more complex (with more variables and constraints) for growing values of H2.

We have also tested the use of the *constrained path* model to derive both the exact model (the equivalent to the HOP model) and the two-phase heuristic of our network design problem. The CPU times were, in general, quite worse. For example, it took almost one day to obtain the optimal solution of problem N50a with H2 = 3 using the HOP model (Table II) but the *constrained path* based model did not obtain any feasible solution within 2 days of CPU time. In the same problem instance, the two-phase heuristic yield the final solution in around 240 minutes while its constrained path counterpart took nearly 1000 minutes. In all the cases where the HOP model did not find the optimal solution, its *constrained path* counterpart was also not able to find the optimal solution and the obtained lower bounds were always worse.

In summary, the two-phase heuristic (based on the hop-indexed approach) is a promising method to handle the design of MPLS over WDM networks with packet level QoS constraints, since it is able to obtain good quality solutions to relatively large problems in realistic computational times.

VI. CONCLUSIONS

In this paper, we have addressed a network design problem that arises in IP networks provisioned through an MPLS network layer operating over a WDM optical network. The network design problem considers the joint determination of the MPLS network layout and the WDM optical layout taking into account both packet level QoS constraints and lightpath level constraints (which accounts for transmission impairments). We proposed an ILP model based on an hop-indexed approach, called the HOP model, to give an exact solution to the network design problem. We have also developed a two-phase heuristic, based on the decomposition of the HOP model in two simpler ILP models. Our results show that the proposed heuristic is successful in producing good feasible solutions for relatively large networks within realistic computational times using the branch-and-bound algorithm. Also, the lower bounds computed through the HOP model show that the two-phase heuristic produces good solutions and, in some cases, achieves the optimal solutions.

ACKNOWLEDGEMENTS

The authors wish to acknowledge the financial support of FCT (Fundação para a Ciência e Tecnologia), Portugal, through project POSI/CPS/41459/2001 MULTIDIM and through grant SFRH/BM/4189/2001.

REFERENCES

[1] X. Xiao, A. Hannan, B. Bailey, "Traffic Engineering with MPLS in the Internet", *IEEE Network*, March/April 2000.

[2] Special issue on "Multiprotocol Label Switching", *IEEE Communications Magazine*, vol. 37, December 1999, pp. 36-68.

[3] A. Parekh and R. Gallager, "A Generalized Processor Sharing Approach to Flow Control in Integrated Services Networks: The Multiple Node Case", *IEEE/ACM Transactions on Networking*, vol. 2, April 1994, pp. 137-150.

[4] B. Mukherjee, "Optical Communication Networks", McGrow-Hill, 1997.

[5] R. Ramaswami and K. N. Sivarajan, "Optical Networks: A Practical Perspective", 2nd edition, Morgan-Kaufmann, 2002.

[6] D. Banerjee and B. Mukherjee, "Wavelength-routed Optical Networks: Linear Formulation, Resource Budgeting Tradeoffs, and Reconfiguration Study", *IEEE/ACM Transactions on Networking*, vol. 8, October 2000, pp. 598-607.

[7] M. Kodialam and T.V. Lakshman, "Integrated Dynamic IP and Wavelength Routing in IP over WDM Networks", *Proceedings of IEEE INFOCOM 2001*, April 2001.

[8] F. Ricciato, S. Salsano, A. Belmonte and . Listanti, "Off-line Configuration of a MPLS over WDM Network inder Time-Varying Offered Load", *Proceedings of IEEE INFOCOM 2002*, June 2002.

[9] A. Balakrishnan, and K. Altinkemer, "Using a Hop-Constrained Model to Generate Alternative Communication Network Design", *ORSA Journal on Computing*, vol. 4, 1992, pp. 192 – 205.

[10] T. Magnanti, P. Mirchandani and R. Vachani, "Modeling and Solving the Two-Facility Capacitated Network Loading Problem", *Operations Research*, vol 43, 1995, pp 142 – 157.

[11] M. Herzberg, S. Bye and A. Utano, "The Hop-Limit Approach for Spare-Capacity Assignment in Survivable Networks", *IEEE/ACM Transactions on Networking*, vol. 3, December 1995, pp 775-784.

[12] L. Gouveia, "Using Variable Redefinition for Computing Lower Bounds for Minimum Spanning and Steiner Trees with Hop Constraints", *INFORMS Journal on Computing*, vol. 10, 1998, pp. 180-188.

[13] L. LeBlanc, J. Chifflet and P. Mahey, "Packet Routing in Telecommunications Network with Path and Flow Restrictions", *INFORMS Journal on Computing*, vol. 11, 1999, pp 188-197.

[14] B. Van Caenegem, W. Van Parys, F. DE Turck and P. M. Deemester, "Dimensioning of Survivable WDM Networks", *IEEE Journal of Selected Areas on Communications*, vol. 16, September 1998, pp. 1146-1157.

[15] M. Tornatore, G. Maier and A. Pattavina, "WDM Network Optimization by ILP Based on Source Formulation", *Proceedings of IEEE INFOCOM 2002*, June 2002.

[16] C. Lopes, A. de Sousa, R. Valadas, "Heuristics for Dimensioning Large-scale MPLS Networks", *in* Internet Performance and Control of Network Systems II, Robert D. van der Mei, Frank Huebner-Szabo de Bucs, Editors, *Proceedings of SPIE Vol. 4523*, 2001.

[17] L. Cardoso, J. Patrão, C. Lopes, A. de Sousa, R. Valadas, "PTPlan MPLS: A Tool for MPLS Network Dimensioning", *Proceedings of 9th Int. Symp. on Modeling, Analysis and Simulation of Computer and Telecommunication Systems* (MASCOTS' 2001), Cincinati, Ohio, USA, August 2001.

[18] L. Gouveia and T. Magnanti, "Network Flow Models for Designing Diameter Constrained Spanning and Steiner Trees", Technical Report, *Centro de Investigação Operacional*, University of Lisbon, W.P. 5/2000, 2000 (to appear in *Networks*).

[19] G. Dahl and L. Gouveia, "On Formulations of the Hop-Constrained Minimum Spanning Tree Problem", Technical Report, *Centro de Investigação Operacional*, University of Lisbon, W.P. 2/2000, 2000.

[20] G. Nemhauser and L. Wolsey, "Integer and Combinatorial Optimization", John Wiley and Sons, 1988.