Gain Compression or Expansion, Distortion and Other Large Signal Power Amplifier Related Phenomena

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Abstract

In this paper, Gain Compression, Expansion and Intermodulation Distortion (IMD) phenomena in power amplifiers is studied and predicted using a new mathematical basis. This type of nonlinear model enabled the design of power amplifiers specially tailored to present a desired IMD versus drive level pattern. Finally, illustrative application examples are presented.

I. INTRODUCTION

One of the major issues in a power amplifier design process is the level of nonlinear distortion allowed, in order to fulfill the specifications of SNR. Normally, to solve this problem, the Input Back-off technique is used. Since that first order procedure consists in lowering input power in order to reduce distortion power at the output, it also decreases output fundamental power, thus degrading efficiency and the overall system performance.

The first objective of this paper is to present a new mathematical model that explains nonlinear distortion mechanisms, which are responsible for the so-called gain compression and expansion phenomena in power amplifiers. This will enable the accurate representation of strange behaviors of the most important distortion effects: 3rd order IMD and harmonic distortion. With this type of information it became possible to understand and therefore possibly overcome some of the associated input back-off problems.

Usual power amplifier's fundamental output power and 3rd harmonic distortion curves versus input power can present two clearly distinct behaviors, as is shown in Fig.1.

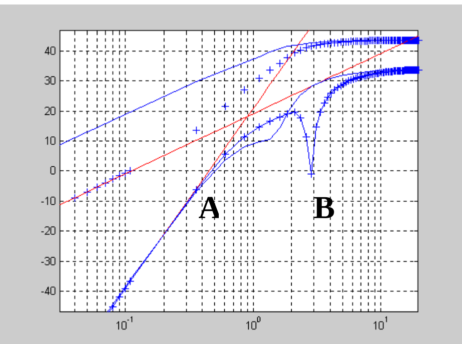

Fig. 1 – First and third harmonics Pin-pout curves

Solid line corresponds to a normal gain compression operation. That means output power first follows linearly the input, rising 1dB for each dB of input power increase. Then, for a determined input drive, it progressively deviates from that curve starting to compress. Beyond this point, the relation between input and output is less than 1dB/dB, tending to an asymptotic rate of 0dB/dB. The corresponding 3rd harmonic distortion can be observed as beginning with a linear behavior too, rising 3dB for each dB of input power. And, when the fundamental starts to compress, it presents a sudden increase, tending again to a constant. So, the ratio between output power and distortion rapidly degrades.

On the other hand, the slashed curve of Fig. 1 represents the case of some times observed gain expansion phenomenon. That means the output fundamental power first linearly follows the input power, then, it starts to get a faster rate of rise (gain expansion), and finally, it compresses again to the above maximum output power. Now, consider the 3rd harmonic curve. It can be seen that first it follows the previous solid curve within the small signal regime, rising 3dB for each dB of input level. Then, contrary the previous case, it presents an unexpected minimum at a determined input power. By observing the two curves, and remembering that the design goal is a certain relation between output fundamental and the 3rd harmonic distortion (C/I), it is no longer evident which input back-off should be imposed. In fact, for the same distortion characteristics, we will get a better output C/I by selecting B as the amplifier working point, than with A.

Other authors [1] have already observed this strange behavior in IMD, but until now no one tried to explain its origins, or predict it without measuring the final amplifier circuit. Since the problem resides on how to predict these different amplifier behaviors, we began by analyzing the amplifier in its small signal regime, and then extended the results to large signal operation. Small signal analysis was based on Volterra Series (VS expansion of the amplifier's active device nonlinear model) [2], while large signal was handled by Describing Function (DF) techniques [3].

In the next section this mathematical approach will be presented. Then, in section III an application example will be shown.

II. MATHEMATICAL APRROACH

If Volterra Series technique [2] is applied to the simplified amplifier nonlinear transfer function of Fig. 2, a good approximation of the characteristic function between points A and B can be obtained with only a 5^{th} degree Taylor series. We called this the small signal analysis, from which output voltage can be given by:

$$Y(\omega)=H_1(\omega)X(\omega)+N_2H_2(\omega_1,\omega_2)X(\omega_1)*X(\omega_2)+ \\ +N_3H_3(\omega_1,\omega_2,\omega_3)X(\omega_1)*X(\omega_2)*X(\omega_3)+\dots \quad (1)$$

Where $X(\omega)$ and $Y(\omega)$ are the frequency domain representation of the input and output signals, respectively, and $H_1(.)$, $H_2(.)$, $H_3(.)$ are the Volterra first 3 nonlinear transfer function of the system, and finally, N_2 and N_3 are constants.

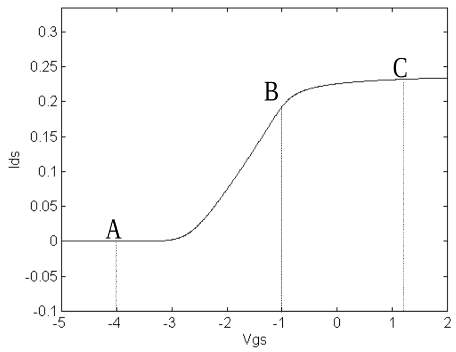

Fig. 2 – Characteristic function

If the input is assumed as a sinusoidal single tone, $X(\omega)=Xe^{j\phi}$, we will have for the 1^{st} and 3^{rd} harmonic behavior:

$$Y(\omega)=H_1(\omega)Xe^{j\phi}+3H_3(\omega,-\omega,\omega)X^3e^{j\phi}+10H_5(\omega,\omega,\omega,-\omega,-\omega)X^5e^{j\phi} \quad (2)$$

$$Y(3\omega)=H_3(\omega,\omega,\omega)X^3e^{j3\phi}+5H_5(\omega,\omega,\omega,\omega,-\omega)X^5e^{j3\phi} \quad (3)$$

As can be seen the 1^{st} harmonic, or the output fundamental power, will rise 1dB for each input (X) dB, until the term $H_3(\omega,\omega,-\omega)X^3$ starts to be important, in comparison to the term $H_1(\omega)X$. So, when $H_3(.)$ is contrary in phase to $H_1(.)$, a compression phenomenon will appear. Or in the opposite case expansion if $H_3(.)$ is in phase to $H_1(.)$. This behavior remains until the term H_5 starts to be important.

Now looking to expression (3), the third harmonic curve will rise 3dB for each dB of input, until the term $H_5(\omega,\omega,\omega,\omega,-\omega)X^5$ starts to be important, and dominates over the term $H_3(\omega,\omega,\omega)X^3$.

If $H_5(\omega,\omega,\omega,\omega,-\omega)$ is in phase with $H_3(\omega,\omega,\omega)$,the third harmonic curve will have an expansion, rising with a slope of more than 3dB for each input dB. Otherwise, if they are opposite in phase, the third harmonic curve will present compression and for a pre-determined input power (X) a minimum can be generated. This minimum will appear at:

$$H_3(\omega,\omega,\omega)X^3e^{j3\phi}=-5H_5(\omega,\omega,\omega,\omega,-\omega)X^5e^{j3\phi} \quad (4)$$

corresponding to an input power of $Pin=\left|\frac{H_3}{5H_5}\right|$.

If a larger input signal is considered, like one that travels far beyond point B, Fig.2, a larger Taylor series degree must be used, in order to maintain Volterra series' accuracy. Then, combining different arrangements of H_i various nulls can be created. It can be proved that circuit's behavior can be very well explained until point B using a simple 5^{th} or 7^{th} Taylor approximation and a Volterra series. But, if the circuit is biased between A and B and excited with a signal that travels to C, more terms of the Taylor series should be used. A very large Taylor series makes the applicability of the Volterra series unrealizable, and so a more powerful analysis tool is needed. In our case we will use the Describing Function (DF) technique [3].

If the signal level is large, only quantitative information of the DF can be retained, loosing all the qualitative information provided by Volterra series. In that case n^{th} harmonic distortion for a single tone input excitation may be represented by:

$$I_{out}(A,n\omega)=\frac{1}{A}\int_{-T}^{T} f_{NL}[v_{in}(t)]e^{-jn\omega t}dt = \\ =\frac{1}{A}\int_{-T}^{T} f_{NL}[A\sin(\omega t)]e^{-jn\omega t}dt$$

; where $v_{in}(t)=A\sin(\omega t)$, and the 3^{rd} harmonic distortion using:

$$DF_{Iout}(A,3\omega)=\frac{1}{A}\int_{-T}^{T} f_{NL}[A\sin(\omega t)]e^{-j(3\omega)t}dt \quad (5)$$

It can be proved, using the expressions above and for a system with a characteristic function as the one presented on Fig. 2. That the third harmonic distortion converges to a constant power with 180º out of phase, (compared with the fundamental output power), when the input power tends to infinity. So, using Volterra series, some knowledge of the third harmonic distortion can be obtained for small signal regimes, and using the Describing Function the large signal regime can be sought.

In conclusion, small and large signal distortion will be integrated in the same mathematical model by using a simplified formula. In the next sub-section this new formula will be presented.

III. SMALL AND LARGE SIGNAL BEHAVIOR MODEL

In order to integrate the small and large signal behavior Volterra series was considered to represent the small signal and the Describing function the large signal, the equivalent formula will be:

$$Y(3\omega)=H_3(\omega,\omega,\omega)X^3e^{j3\phi}+5H_5(\omega,\omega,\omega,\omega,-\omega)X^5e^{j3\phi}+LS(3\omega) \quad (6)$$

for a single tone excitation, where LS(3ω) is the now called Large-Signal Contribution:

$$LS(3\omega)=DF_{Iout}(A,3\omega)-Y(3\omega) \quad (7)$$

LS(3ω) is near zero, compared with the Volterra series, between A and B (if the circuit is biased between A and B) and is very large when the signal travels far from B towards C.

Using $DF_{Iout}(A,3\omega)$ it can be proved that the output distortion power of the 3rd harmonic [Y(3ω)] tends to a constant, and its phase to 180º, as explained above. So, if Y(3ω) has a phase of 0º in its small signal region, and then tends to 180º for large signal, it must have a zero that results from the iteration between LS(3ω) and the so called small signal behavior.

It can also be proved that small signal behavior has a strong relation with the characteristic function derivatives [4]. So, if the derivatives of the characteristic function are known, then the device must be biased in a point where its small signal 3rd order distortion output has a 0º phase in order to justify a minimum at a certain output power. This minimum will appear because the large signal phase tends to 180º

Although this mathematical derivation was developed to a single tone excitation, it can be proved that it can be applied to any type of input excitation.

Looking back at (6), can be seen that it is possible to generate multiple nulls. Consider for example that the circuit is biased between point A and B, where $H_3>0$ >>>H_5, (positive third derivative). Then, a null will be generated by the iteration of H_3 and LS, because the phase of Y(3ω) tends to 180º for large signal. This null will be generated when the time domain excursion of the excitation signal approaches point B, as LS suffers there a rapid grow. Because H_3 is positive, in-band distortion that falls over the fundamental will add to H_1, and so an expansion phenomenon will appear - dashed lines of Fig.1.

The second case is where $H_3<0$ >>>H_5, (negative third derivative). That means LS and H_3 are always in phase, and so, as they will add up, nulls are no longer possible. Also, because $H_3<0$, it will subtract to H_1 and a compression phenomenon is present - solid lines of Fig.1.

Consider now that the circuit was biased near the 3rd order IMD sweet-spot where $H_3=0$. Now, small signal behavior will depend only on H_5, and the same conclusions can be taken for $H_5>0$ or $H_5<0$ and H_5>>>H_7.

Finally, when $H_3\neq 0 < H_5$ we can have a small signal null if H_3 is opposite in phase to H_5, and a large signal null if $H_5>0$.

In the next section, some illustrative examples will be proposed, in order to prove its applicability to real devices and systems.

IV. APPLICATION EXAMPLE

In order to prove the practical validity of the above-explained theory, a MESFET power amplifier with a characteristic function similar to that of Fig.2 was biased in several conditions.

The amplifier schematic is presented in the next figure:

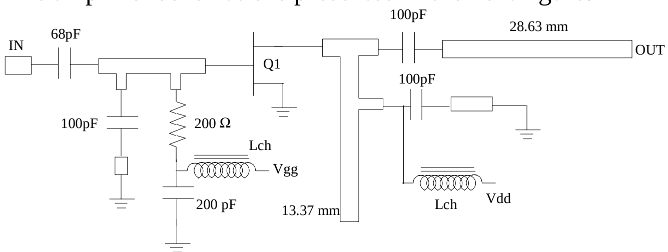

Fig. 3 – Schematic diagram of the power amplifier circuit.

Both, the amplifier characteristic function and that function's derivatives were extracted using a harmonic balance simulator, and are presented next.

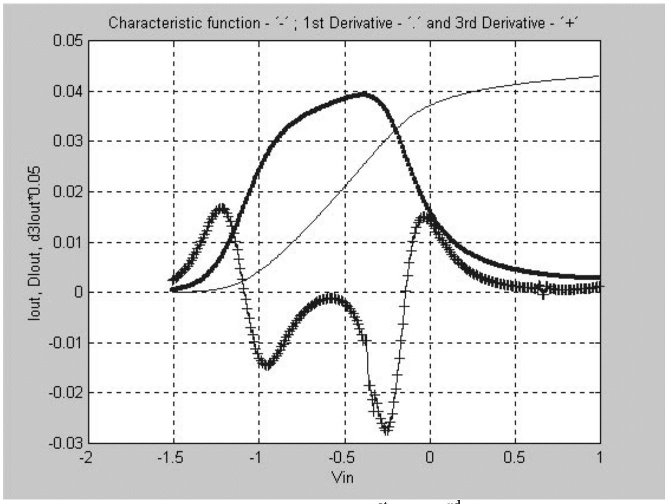

Fig. 4 – Characteristic function, 1st and 3rd derivatives.

Using [4], the relation between the Volterra series and the nonlinear characteristic function derivatives can be taken. So, if some bias points are chosen, we can generate minimums at certain input power excitation.

In order to prove this, we excited the power amplifier with a two tone signal, and used two different bias points. One at the maximum positive third order derivative, class C, and the other as a standard class A power amplifier. In the next figures these different behaviors can be observed in the fundamental output frequency and at the IMD excitation.

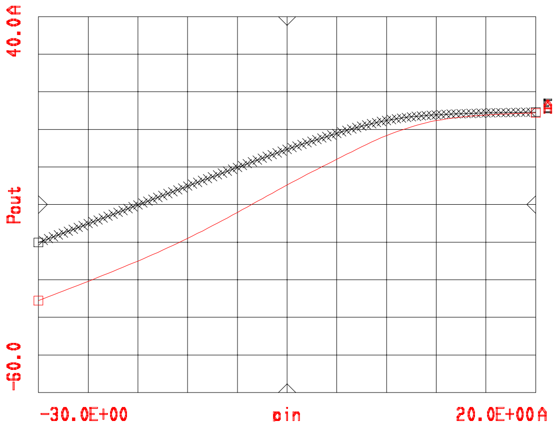

Fig. 5 – Output Power for two bias points, Class A(+) and Class C.

As can be seen from figure 5, the fundamental output power for the class A amplifier compresses in all the input sweep range, while the fundamental output power for the class C amplifier, starts to present an expansion behavior, tending to the same compressed output power. These results are well predicted by the above formulas presented in this paper.

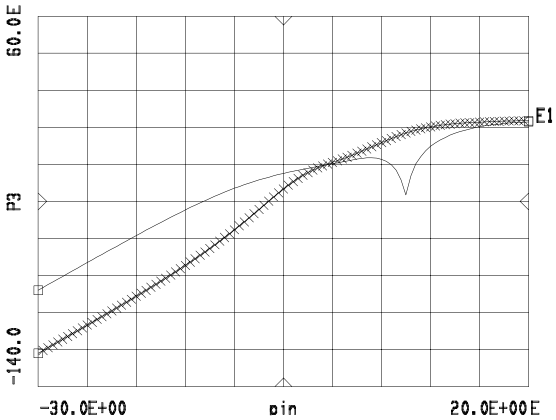

Fig. 6 - IMD Power for two bias points, Class A(+) and Class C.

From Fig. 6, it is possible to see that the class A power amplifier IMD rises 3dB for each input dB in its small signal region, and then, near the 1 dB compression point, starts to compress, having a sudden increase degrading overall IMD. In contrast, the Class C amplifier IMD rises 3dB for each dB of input power, and then, near the 1 dB compression point, a minimum can be observed, like the one predicted by the mathematical formula.

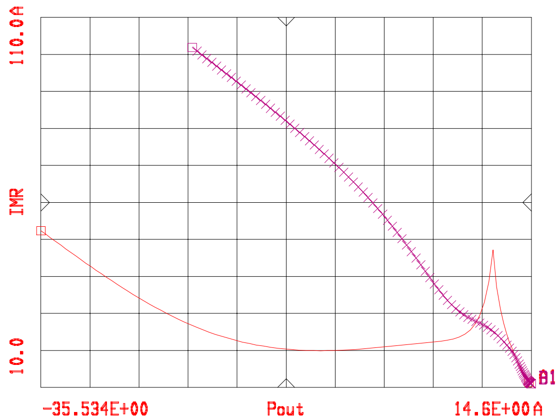

Fig. 7 - IMR for two bias points, Class A(+) and Class C.

In figure 7, the IMR (Intermodulation ratio, IMR=Pout-IMD) is plotted versus the output power. There, it is possible to see that for an equivalent output power near 10 dBm the IMR is better for the class C power amplifier. Despite the better small signal IMR presented by class A, in the large signal regime, it is still possible to generate a better IMR for the class C amplifier. In summary, for a certain IMR specification it is possible to use a Class-C amplifier, taking all its recognized advantages in power added efficiency, in applications where traditional designs would advise the more obvious linear Class-A.

V. CONCLUSIONS

In conclusion, this paper shows, for the first time, a rigorous formula to predict and to explain the so-called IMD large signal sweet spots. Using this type of simplified formulation, it is possible to predict compression expansion and distortion behaviors, not only in the small signal regimes, but also in the large signal regimes.

It was proved that by using an a priori more nonlinear amplifier, like a class C designs, it is possible to have a better distortion behavior in the large signal regime, where the class C amplifier presents a better power efficiency, and is normally used.

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