Abstract — this paper presents advancement in Power Amplifier, PA, distortion evaluation through the system figure of merit Error Vector Magnitude, EVM. Previous studies had quantified the EVM in a memoryless device and related to Signal-Noise Ratio, SNR. With the increase in the bit per hertz relation and amplitude variations in the modulated waveform found in the present day transmitters give rise to distortion thus a decrease in the SNR and new kind of PA effects start playing an important role with relation to the memoryless case. As result there is give great importance to accounting for all PA effects on the EVM. So in this paper we quantify the EVM in a Wiener-Hammerstein, WH, system model, which will give a better accuracy on the SNR estimation for the real PA behavior.

I. INTRODUCTION

The possible relations between EVM and RF/Wireless components figures of merit is of primordial importance for improving the system specifications between the RF circuit design engineer and the RF/Wireless system design engineer. The understanding of these relationships allows the RF circuit design to be able to make some compromises without sacrificing the system behavior of the overall scenario. This is why there are several papers that deal with this problem in the past, by allowing a precise study of the relation between typical figures of merit of RF devices as Interception point of third order, IP3, phase noise, noise figure, etc and EVM.

The most important relation established with EVM is with SNR, which is the principle figure of merit for any electronic system and is given by:

\[
EVM = \frac{1}{\sqrt{SNR}}
\]  

Expression (1) is as, or even more important having in mind (2) where we can relate SNR with the system distortion level, \( S_{\text{uncorr}} \):

\[
SNR = \frac{S_{\text{corr}}}{S_{\text{uncorr}}}
\]  

Where \( S_{\text{corr}} \) and \( S_{\text{uncorr}} \) are the correlated and uncorrelated parts of the spectrum respectively.

So in this work we pretend to quantify the effect of a RF PA with all non-idealities in the Error Vector Magnitude.

The previous work [1], has only taken into account a simple memoryless device model to characterize the PA behavior which imposes a simple amplitude compression/expansion and does not take into account the phase rotations resulting from the real PA effects.

The demands on the present wireless services require an increase of the bit rates which associated to a limited spectrum available leads to an increase of bit per hertz relation and consequently an SNR reduction. Therefore effects, until now uncounted for, start playing an important role on the system behavior, magnifying the importance of quantifying them. In order to fully characterize the RF device for system level effects, it is necessary to use the WH model [2]. Therefore in this paper we go a step further and obtain the relation between EVM and the WH model parameters.

We also perform this analysis obtaining a expression which establishes a direct relationship between EVM and the system parameters giving the understanding of how and how much these parameters effect the EVM.

To make the simulation we use the complex envelope method without loss of generality which allows an easier and faster analysis and simulation respectively [2].

II. ERROR VECTOR MAGNITUDE REVISITED

EVM is a figure of merit that quantifies the quality of digital modulated signals, and is defined by the following formula:

\[
EVM = \sqrt{\frac{\sum_{n=0}^{+\infty} \left( X_i(nT) - x_i(nT) \right)^2 + \left( X_q(nT) - x_q(nT) \right)^2}{\sum_{n=0}^{+\infty} \left( X_i^2(nT) + X_q^2(nT) \right)}}
\]  

Where \( (X_i,X_q) \) are the ideal in-phase and quadrature components respectively, \( (x_i,x_q) \) are the correspondent optimal output sampling position measured values, and T the bit period. EVM is a figure of merit which is signal independent, therefore its blind to baseband pulse shaping format, envelope variations or even pre-distortion mechanisms. This is so because it only quantifies the optimal demodulated sampling moments.
EVM can be increased due to amplitude and phase changes and is originated by linear and nonlinear phenomena, although linear deviations can be viewed as power scales.

EVM is the mean of the total power deviation, in cases where the system do not introduce band limitations all symbols will suffer the same effects independently of the bit sequence, thus every symbol has the same deviation, obviously proportional to its complex envelope power [3]. In this case the deviation to the ideal constellation is only due to distortion, and so EVM will be a good measure of the system distortion, through expressions (1) and (2).

In the above condition we only have to calculate each different constellation symbol for the entire observation interval once. The symbols that have different complex power envelopes are considered different symbols.

Looking briefly to (3) and remembering the definition of a complex power envelope signal for one symbol:

\[ \tilde{P}_{\text{in}} = X_i^2 + X_q^2 \]  

Expression (3) can be seen as the square root of the normalized power deviation from each measured symbol to the equivalent ideal symbol.

Where the tilde is used to indicate a complex envelope. The relation between the vector distance and the difference of two complex envelope power signals is given by:

\[ \text{vector} = \sqrt{(X_i - x_i)^2 + (X_q - x_q)^2} = \sqrt{P_{\text{error}}} \]  

Where

\[ \sqrt{P_{\text{in}}} = \sqrt{X_i^2 + X_q^2} \]
\[ \sqrt{P_{\text{out}}} = \sqrt{x_i^2 + x_q^2} \]

We can represent this in Fig. 1.

\[ (x_i, x_q) \quad \tilde{P}_{\text{in}} \quad \sqrt{P_{\text{in}}} \quad \sqrt{P_{\text{error}}} \quad \sqrt{P_{\text{out}}} \quad (X_i, X_q) \]

So (3) can be rewritten as follows:

\[ EVM = \frac{\sqrt{P_{\text{out}}} - \sqrt{P_{\text{in}}}}{\sqrt{P_{\text{in}}}} = \frac{\tilde{P}_{\text{error}}}{\sqrt{P_{\text{in}}}} \]  

Expression (6) is valid only for systems which don’t introduce phase shift in the constellation, thus this approach is valid only for systems that preserve the phase of the input complex envelope signals, like memoryless devices which only consider amplitude variations for \( \sqrt{P_{\text{out}}} \) along the \( \sqrt{P_{\text{in}}} \) vector direction. Fig. 3 demonstrates a system which introduces phase changes, therefore (6) is no longer valid. This problem will be dealt with in the section where the model WH model is considered.

III. IMPACT ON EVM DUE TO A LTI FILTER

A Linear Time Invariant, LTI, filter for the in-band frequencies can be simplified by its low-pass equivalent, LPE, transfer function in the frequency domain as:

\[ H(w) = |H(w)| \cdot e^{-j[\theta(w) + \Delta \theta]} \]  

Where \( |H(w)| \) is the filter attenuation, \( \theta(w) \) the linear part of the phase which varies linearly with frequency and \( \Delta \theta \) the phase offset which introduces a phase shift in the in-band frequencies. The attenuation and the phase shift will increase the EVM, introducing a compression and a rotation in the constellation diagram respectively, and these effects will be independent of the signal form.

![Fig. 2. Amplitude and phase response of the filter LPE](image_url)

In order to quantify the effect imposed by the RF filter in the EVM we apply the low-pass equivalent transformation to the filter transfer function, which consists in the translation of the RF spectrum to base band using the follow procedure [2]:

1 We consider that \( X_i \) uncorrelated with \( X_q \).
1. Decompose the transfer function $H(w)$ using the partial fraction expansion.
2. Discard the poles that are located on the negative-frequency half-plane.
3. Shift the poles located on the positive-frequency half-plane to the zero axis by substituting $z \rightarrow z + e^{j\theta}$.

Looking to the phase plot in Fig. 2 it can be seen that for DC the phase is different from zero by an amount $\Delta \theta$, so taking the phase in DC we obtain the phase shift responsible for the constellation rotation. For the attenuation we perform the in-band amplitude mean value of the filter low-pass equivalent.

In practice, to improve the computation efficiency, we can excite the filter with a test signal and then apply the low-pass equivalent to the input and output signal in order to extract the phase shift and attenuation, as using (8) and (9) respectively.

$$\Delta \theta = \angle \max (R_{xx}(\tau))$$ (8)
$$\text{att} = \frac{\text{Re} \{ \max (R_{xx}(0)) \}}{\text{Re} \{ \max (R_{xx}(0)) \}}$$ (9)

Where $R_{xy}(\tau)$ is the cross correlation between signal $x$ and $y$. For precise phase shift estimation we must compensate the attenuation introduced in the output signal.

The test signal can be any signal, as a QPSK modulated waveform, or the usual and easy to use laboratory two-tone test signal. It is important to note that the precision which determines the filter parameters is essential to the correct prediction of the device behavior.

IV. RELATE EVM TO IP3

In this section we will quantify the influence of a memoryless nonlinear power amplifier in EVM, other authors [1] have already performed the same analysis but in the frequency domain, here we use a time domain approach which gives a better sensibility on how the different system parameters influence the system response. To describe the memoryless PA behavior we use the mathematical model presented in (10) to simplicity we truncate the nonlinear response to the third order, this is enough to get a complete idea of the device influence on the EVM.

$$y(t) = a_1 \cdot x(t) + a_2 \cdot x(t)^2 + a_3 \cdot x(t)^3$$ (10)

Where $a_1$ is the linear gain, $a_2$ and $a_3$ the second and third order nonlinear coefficients, $x(t)$ is the input RF digitally modulated signal and $y(t)$ RF output signal.

The even order terms generate components at baseband and at even order harmonics of the carrier frequency, so $a_2$ could be rejected since we only pretend in-band components. We will also quantify $a_1$ as being part of the signal therefore we just want to measure the impact caused by the distortion, making the EVM a measure of the system distortion level.

The memoryless output complex envelope signal is given by:

$$\check{y}(t) = (X_j(t) + j \cdot X_q(t)) \left[ a_1 + \frac{3}{4} \cdot a_3 \cdot (X_j^2(t) + X_q^2(t)) \right]$$ (11)

Where $X_j(t) + jX_q(t)$ are the complex envelope of $x(t)$.

To obtain EVM we sampled (11), in the optimal instants, leading to the discrete form (12):

$$\check{y}(k) = \sum_{k=-\infty}^{\infty} \left[ (X_j(k) + j \cdot X_q(k)) \cdot \left[ a_1 + \frac{3}{4} \cdot a_3 \cdot (X_j^2(k) + X_q^2(k)) \right] \right]$$ (12)

This result shows when a digital modulated signal with a constant time domain complex envelope like a QPSK signal is submitted to a memoryless device (which can be modeled by a polynomial response), only a linear deviation in the constellation diagram (a power scale) is introduced. Therefore there is not EVM degradation, or all distortion introduced by the nonlinear device are correlated.

Rewriting (3) we have:

$$EVM = \sqrt{\frac{\sum_{k=1}^{N} \left[ (X_j(k) - X_j(k) \cdot \text{cte}(k)) \right]^2 + \left( X_q(k) - X_q(k) \cdot \text{cte}(k) \right)^2}{\sum_{k=1}^{N} [X_j^2(k) + X_q^2(k)]}}$$ (13)

With:

$$\text{cte}(k) = \left[ a_1 + \frac{3}{4} \cdot a_3 \cdot (X_j^2(k) + X_q^2(k)) \right]$$

Considering that $a_i$ is part of the desired signal

$$EVM = \sqrt{\frac{\sum_{k=1}^{N} \left[ (X_j(k) \cdot \left( a_1 - \text{cte}(k) \right)) \right]^2 + \left( X_q(k) \cdot \left( a_1 - \text{cte}(k) \right) \right)^2}{a_i^2 \cdot \sum_{k=1}^{N} \tilde{P}_m(k)}}$$ (14)

With $\tilde{P}_m(k) = X_j^2(k) + X_q^2(k)$, and $k = l, .., N$

Where $N$ is the number of different constellation symbols, and so (14) is a sum for each different constellation symbol. As an example, in a QPSK signal we just have to perform one symbol since they all have the same complex envelope power.
For a 16-QAM we have to calculate three symbols, where one of which has the double probability of occur due to the existence of two symbols in the constellation having the same complex envelope power.

We can also establish a relationship with the helpful figure of merit IP3. From [4] we know that IP3 is given by:

\[ IP3 = \frac{2}{3} \frac{a_1^3}{a_3} \]  

(15)

So the EVM can be related with IP3 as follow:

\[ EVM = \sum_{k=1}^{N} \frac{a_1^2}{2 \cdot IP3} \tilde{P}_{in}(k) \]  

(16)

V. EXTRACT EVM FOR A WIENER-HAMMERSTEIN MODEL

The WH model is composed by the cascade: pass band filter, polynomial memoryless nonlinearity, and pass band filter, respectively, Fig. 3. This causes a phase rotation (due to filters phase shift) beyond the known amplitude expansion and compression phenomena.

To quantify the degradation introduced in the EVM by the WH model, we use the expressions shown in (8), (9) and (11), with the parameters for the first filter, \((\Delta \theta_1, att_1)\), the amplifier, \((a_1, a_2)\), and the second filter, \((\Delta \theta_2, att_2)\).

\[ x(t) \rightarrow \text{Amplifier} \rightarrow y(t) \]

Fig. 3. Wiener-Hammerstein model

To explain the procedure, a QPSK (4-QAM) signal, \(k = 1\) was used. However this method can be directly extended to a more complex diagram constellations, as M-QAM with M greater than 4, performing only the sum of the several symbols with different complex envelope powers.

The analysis starts by making \(x(t)\) pass through the first filter with parameters \((\Delta \theta_1, att_1)\), which leads to the following complex envelope components:

\[ \tilde{x}_11 = att_1 \cdot \sqrt{x_1^2 + x_q^2} \cos(\theta - \Delta \theta_1) \]

\[ \tilde{x}_1q = att_1 \cdot \sqrt{x_1^2 + x_q^2} \sin(\theta - \Delta \theta_1) \]  

(17)

The first part refers to a compression due to the filter attenuation and the second to the phase shift, where:

\[ \theta = \arctg \left( \frac{X_q^2}{X_i^2} \right) \]  

(18)

With the first RF filter effect quantified, \(x_1\), we are able to apply (11), and analyze the memoryless device effect. So we obtain:

\[ \tilde{x}_2 = (x_1 + j \cdot x_q) \cdot \left[ a_1 + \frac{3}{4} a_3 \cdot att_1^2 (X_i^2 + X_q^2) \right] \]  

(19)

Finally \(x_2\) will pass through the second filter and the output signal \(y(t)\) is obtained:

\[ \tilde{y}_i = \left[ a_1 + \frac{3}{4} a_3 \cdot att_1^2 \cdot \tilde{P}_{in} \right] \cdot att_2 \cdot \sqrt{x_1^2 + x_q^2} \cos(\theta - \Delta \theta_1 - \Delta \theta_2) \]

\[ \tilde{y}_q = \left[ a_1 + \frac{3}{4} a_3 \cdot att_1^2 \cdot \tilde{P}_{in} \right] \cdot att_2 \cdot \sqrt{x_1^2 + x_q^2} \sin(\theta - \Delta \theta_1 - \Delta \theta_2) \]  

(20)

Now we are able to quantify the degradation introduced by the WH model in the EVM through (21), where:

\[ \tilde{P}_{in} = x_i^2 + x_q^2 \]

It’s convenient to remember again that (21) can be extended for every digital modulation scheme, being only necessary to calculate all different constellation symbols resulting (21) under the square root a summation of the different constellation symbols.

VI. SIMULATIONS

In order to validate the performance of the approximations proposed above, we use SIMULINK [5] and design a transmitter based on a CDMA reverse-link using a 16-QAM modulation scheme. The filters have two times the signal bandwidth, and the amplifier is described by a third order non linear polynomial model, presenting gain compression. To perform EVM we get the complex envelope of the RF output signal and the ideal based and I/Q sequence, applying in (21).

The filters’ parameters are obtained using the procedure explained above, with two signals captured with probes, one at the input and the other at the output of the filters.

In Fig. 4 the transmitter is presented with the WH model for the RF circuit.
Fig. 4. QPSK transmitter architecture

Fig. 6 validates the proposed approximation by confirming an excellent approximation between the simulated EVM values and (21), we also include the PA AM/AM curve, to give an idea of the PA operating point, Fig. 5.

VII. CONCLUSIONS

In this work, an analysis of phase and amplitude distortion introduced by the PA is evaluated through EVM and thus the SNR, a very accurate estimate is obtained.

A step has been taken in the understanding of a real PA effect on digitally modulated signals allowing a higher accuracy on the distortion measurement.

The next step is to include long term memory effects in this analysis.

REFERENCES


[5] Simulation and Model-Based design, Matlab tool

Fig. 5. System metrics. o : AM/AM

Fig. 6. 16-QAM, EVM approximation; o : simulated; and * : estimated.