Abstract — This talk will discuss the problem of microwave characterization of a broad class of nonlinear dynamic microwave devices and systems. For that, it will be shown how the amplitude and phase responses to two-tones or, more general multi-sines, can be accurately captured, and related to each other in practical PA cases. The characterization of nonlinear mixing products of microwave PAs, evidencing nonlinear memory effects, will be used as an illustrative example of the proposed nonlinear measurement techniques.

I. INTRODUCTION

RF and Microwave measurements have been devoted to single frequency excitations, what is normally called one tone measurements, from several years till now. Nevertheless this excitation is interesting for characterizing linear systems, but on nonlinear systems they can only represent harmonics of that signal, and definitely they cannot represent in-band distortion, or excite base band components. This fact as drove microwave engineers to use higher order and higher complex excitation signals, as two-tone sinusoidal excitations [1], multi-sine excitations [2], and more recently to chirp two tone signals [3] and/or non-uniform multi-sine excitations [4].

The demanding for this type of newly excitation signals is mainly driven by the fact that newly developed technologies need to be represented by behavioral models that mimic dynamic effects, been those long term memory and/or short term memory. The full understanding of the DUT’s dynamic effects is of paramount importance in modern communication systems since the occupied bandwidth and modulation method’s complexity are dramatically increasing. These dynamic effects can be divided into short and long term, with short and long referring to the time constants involved in the impulse response tail of a nonlinear dynamic system. The long term memory time constants impact the signal’s envelope, while the short time constants affect the RF signal. Since in a communication system the information is carried by the envelope, the understanding of the long term memory effect mechanisms is a fundamental topic for understanding the system’s performance degradation [5].

The characterization of these effects demands for an excitation signal that is able to represent the envelope performance. That is the reason why RF engineers have moved to two-tone excitation tests for their DUT characterization. Despite that, two-tone excitation only excite a single base-band frequency, which drove again RF engineers to design better excitation signals that allow the excitation of a large number of base band frequencies, those include burst signals [3] and/or multi-sines [2]. Nevertheless some questions remains on the equivalence of those two, that is, the equivalence of burst two-tone excitation and multi-sine behavior. Since in a two tone excitation we only excite one base band frequency for each test, while in a multi-sine excitation we excite a huge number of base band frequencies simultaneously.

In this paper we present the relationships between two-tone excitation and multi-sine excitation for a nonlinear third order dynamic system, and present a measurement bench that is able to calculate and obtain a multi-sine description by using two tone measurements.

II. BEHAVIORAL MODEL OF A THIRD ORDER NONLINEARITY PRESENTING MEMORY

Nonlinear DUT’s presenting dynamic effects, can in most practical cases be approximated by a third order nonlinearity. For systems that respond to a narrowband signal, that third order nonlinearity can be decomposed as the sum of a cubic polynomial direct path response, with an up-converted base-band component. That base-band component is the response of a second-order nonlinearity further pressed with memory in a low-pass filter that mimics the base-band response of the nonlinear system, Fig. 1, [5, 6].

The in-band intermodulation distortion output for a two-tone signal is further given by [5, 6]:

\[ y(t) = H_x(x(t) + x(t)) + H_y(x(t) - x(t)) + \ldots \]
\[ H_3(\omega_2, \omega_2, -\omega_3) = K_3 - K_2 [2H_2(\omega_2, -\omega_3) + H_2(\omega_2, \omega_2)] \]  
\[ (1) \]

where \( K_2 \) and \( K_3 \) are the second and third order coefficients controlling the in-band nonlinear distortion, respectively, while \( H_2(\omega_2, -\omega_3) \) and \( H_3(\omega_2, -\omega_3) \) are the 2nd order nonlinear transfer functions responsible for the base-band and second harmonic signal components.

If a multi-sine is considered, then the output distortion is similar to what can be seen in Table 1, where these components can be considered constant since the relative bandwidth change with the tone spacing is very small.

### Table 1 – 5 tone third order mixing products.

<table>
<thead>
<tr>
<th>Spectral Regrowth tone</th>
<th>( \omega_a )</th>
<th>( \omega_b )</th>
<th>( \omega_c )</th>
<th>( \omega_d )</th>
<th>( \omega_e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixing components</td>
<td>( \omega_a + \omega_b - \omega_c )</td>
<td>( \omega_a + \omega_b - \omega_d )</td>
<td>( \omega_a + \omega_b - \omega_e )</td>
<td>( \omega_a + \omega_c - \omega_d )</td>
<td>( \omega_a + \omega_c - \omega_e )</td>
</tr>
</tbody>
</table>

If we consider the spectral regrowth tone identified as \( \omega_4 \), we can see that the output signal depends on:

\[ H_3(\omega_2, \omega_2, -\omega_2) = K_3 - K_2 [2H_2(\omega_2, -\omega_2) + H_2(\omega_2, \omega_2)] \]  
\[ (2) \]

From Eq. (2) we see that the response of the system for a two-tone signal with tone spacing \( \omega_2 - \omega_3 \) is similar to the multi-sine case.

This formulation will allow us to derive formulas that relate the two-tone and multi-sine excitations very efficiently and in a straight way.

### III. Characterization of Each Multi-sine Distorted Tone

For the calculation of each distorted tone contribution, a two-tone test is performed and the result is computed according to Eq. (1).

So we will come out with a table that will contain each value of \( F_2(\omega_2, -\omega_3) \), and the constant part \( K \), which is calculated from the asymptotic behavior of \( Y(2\omega_2, -\omega_3) \) at very low frequency separations, ideally zero Hz. So the two-tone contribution becomes:

\[ Y(2\omega_2, -\omega_3) = [K - 2F_2(\omega_2, -\omega_3)]X(\omega_2)X(\omega_3)X(-\omega_3) \]  
\[ (4) \]

where \( K \) is:

\[ K = k_1 - k_2H_2(\omega_1, \omega_3) - 2k_2H_2(\omega_1, -\omega_3) \]  
\[ (5) \]

and \( F_2(\omega_2, -\omega_3) \) is the term that varies with tone spacing.

Changing the tone spacing, different values of \( F_2(\omega_2, -\omega_3) \) are computed and stored in our tone separation table. Since the terms \( F_2(\omega_2, -\omega_3) \) in the multi-sine case, add in voltage, we must have them characterized both in amplitude and phase.

So, for each frequency component of the two tone excitation we need to solve the following equation:

\[ H_3(\omega_1, \omega_3, -\omega_3) = K - 2F_2(\omega_1, -\omega_3) \]  
\[ (6) \]

In a test with arbitrary number of tones, the system to be solved can be represented in a matrix form as:

\[
\begin{bmatrix}
1 & 0 & \ldots & 0 \\
1 & 2 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots \\
1 & 0 & \ldots & 2
\end{bmatrix}
\begin{bmatrix}
K \\
F_2(\Delta\omega) \\
\ldots \\
F_2[(n-1)\Delta\omega]
\end{bmatrix}
= 
\begin{bmatrix}
H_3^{(k)}(\omega)/3 \\
H_3^{(\Delta\omega)}(\omega)/3 \\
\ldots \\
H_3^{(n-1)\Delta\omega}(\omega)/3
\end{bmatrix}
\]  
\[ (7) \]
The measurement bench now proposed allows the characterization of the third order transfer functions either in amplitude, but also in phase as was previously presented in [7, 8].

IV. MULTI-SINE EXPERIMENTAL RESULTS

In order to clearly obtain the response of a multi-sine excitation using two-tone characterization, a setup which comprises a low power amplifier was built, the amplifier could be tuned in different arrangements, so we can generate dynamics by merely change the bias networks.

The test was run at a central frequency of 1 GHz with a tone spacing of 20 kHz. The input signal is composed by 20 tones leading to a 400 kHz bandwidth signal. A record of 1000 waveform segments with random phase was used as a random multi-sine signal to predict the output of the system to a narrow band Gaussian noise input.

A test with two tones was then performed, considering an arrangement of different two-tone excitations for $\Delta \omega = 20\text{KHz}$, $40\text{KHz}$, $60\text{KHz}$, $80\text{KHz}$, $100\text{KHz}$, $120\text{KHz}$, $140\text{KHz}$, $160\text{KHz}$, $180\text{KHz}$ and $200\text{KHz}$.

This test was done first to a PA in which the bias network presents a varying base band characteristic.

In Fig.5 the measured results for the multi-sine distortion are presented with the results arising from our method superimposed on the same Fig.

A good agreement can be noticed between the computed values and the measured output.

The increasing error observed in the low power tones, was attributed to the fifth order distortion that is always
present in a real system but was not accounted for in our third order analysis.

Next test includes a bias network that imposes a memoryless behavior to the amplifier, Fig. 6 presents the obtained results.

A good agreement is obtained even in this ideal case stating the applicability of the method.

Fig. 6 – Measured output and predicted response for the memoryless amplifier.

Finally a strange case was measured where the bias network has a strong impact on the nonlinear distortion, the output is presented in the Fig. 7.

Fig. 7 – Measured output and predicted response for the amplifier presenting memory and asymmetry.

The predicted response and the measured output are in perfect agreement.

The memory effects can also be noticed in the co-channel distortion in the shape of the correlated and uncorrelated distortion components, which deserves a special attention, since they state that co-channel distortion and adjacent channel distortion are in fact very different from each other.

V. CONCLUSION

In this paper a formal and easy way to extrapolate multi-sine figures of merit from simple two tone measurements was presented.

This is believed to be a step forward in nonlinear behavioral model extraction from two tone measurements.

ACKNOWLEDGEMENT

This work was partially supported by the EU under the Network of Excellence–TARGET contract IS-1-507893-NoE and Project ColteMepai.

REFERENCES


