Multi-sine Response of Third Order Nonlinear Systems with Memory Based on Two-tone Measurements

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***Abstract* — This paper presents a framework that allows the third order response computation of a dynamic nonlinear system to a multi-sine signal input from only standard two-tone test results. A simulation implementation of the methodology is presented for a third order system with memory. The validation test is performed with a five tone uncorrelated input signal and states the feasibility of the method in the intermodulation distortion's prediction. The work now presented is a step forward to the understanding of the memory generation mechanisms and in the extrapolation of the usual standard RF test results to the prediction of the dynamic system's output to a multi-sine signal excitation.**

I. Introduction

The full understanding of power amplifier's memory effects is of paramount importance in modern communication systems since the occupied bandwidth and modulation method's complexity are dramatically increasing. These memory effects can be divided into short and long term, with short and long referring to the time constants involved in the impulse response tail of a nonlinear dynamic system. The long term memory time constants impact the signal's envelope, while the short time constants affect the RF signal. Since in a communication system the information is carried by the envelope, the understanding of the long term memory effect mechanisms is a fundamental topic for understanding the system's performance degradation.

It has already been proved that the long term memory time constants can be attributed to the low frequency behavior of the PA, i.e., from the bias matching networks, device thermal response and trapping effects [1]. Despite the relevance of these results, they always address two-tone excitation tests, while nothing is said about the mapping of those results into multi-sine excitation prediction.

One method usually accepted to study the memory impact of nonlinear systems is based on sweeping the tone spacing in a two-tone IMD test. Indeed, the variation of the intermodulation distortion with the tone spacing is an obvious symptom of long term memory effects [2]. Nevertheless, the link of those results with the ones obtained with a real signal, or with a multi-sine signal was not addressed until now.

In [3], a relation between two-tone and multi-sine signal tests was presented for a memoryless third order nonlinearity. Then, it was afterwards extended for fifth order systems in [4]. Nevertheless, both of these analyses consider memoryless nonlinearities, and no information is given on how the two-tone memory effects are related to the memory effects appearing in a more complex (and closer to real wireless excitations) multi-sine signal.

In this paper we propose a first study for mapping the distortion observed under two-tone test's onto the one obtained with a multi-sine uncorrelated signal, within a nonlinear third order dynamic system.

II. Response of a Third Order Nonlinearity Presenting Memory

The nonlinear in-band response of a third order nonlinear system with memory, to a narrowband signal, can be decomposed as the sum of a cubic polynomial direct path response, with an up-converted base-band component. That base-band component is demodulated from the RF signal in a second-order nonlinearity and then pressed with memory in a low-pass filter that mimics the base-band response of the nonlinear system, Fig. 1, [5, 6].

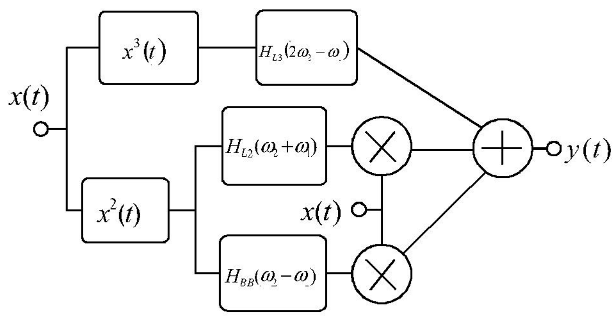

Fig. 1 – Third order dynamic nonlinearity model.

According to this model, the in-band intermodulation distortion output for a two-tone signal is given by [2,5]:

$$H_3(\omega_2,\omega_2,-\omega_1) = K_3 - K_2\left[2H_2(\omega_2,-\omega_1) + H_2(\omega_2,\omega_2)\right] \quad (1)$$

where K_2 and K_3 are the second and third order coefficients controlling the in-band nonlinear distortion, respectively, while $H_2(\omega_2,-\omega_1)$ and $H_2(\omega_2,\omega_2)$ are the 2[nd] order nonlinear transfer functions responsible for the base-band and second harmonic signal components. If we now consider an uncorrelated multi-sine excitation, the output distortion from a nonlinear dynamic system will be an addition in power of several components depending on the tone spacing. Table 1 presents these components obtained for a five-tone signal (see also Fig. 2).

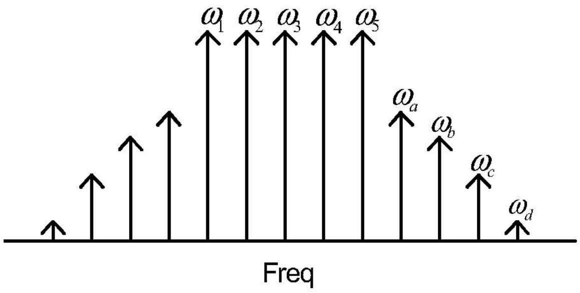

Fig. 2 – Five tone multi-sine and correspondent IMD distortion

Spectral Regrowth tone	ω_a	ω_b	ω_c	ω_d
Mixing components	$\omega_5+\omega_5-\omega_4$ $\omega_5+\omega_4-\omega_3$ $\omega_5+\omega_3-\omega_2$ $\omega_5+\omega_2-\omega_1$ $\omega_4+\omega_4-\omega_2$ $\omega_4+\omega_3-\omega_1$	$\omega_5+\omega_5-\omega_3$ $\omega_5+\omega_4-\omega_2$ $\omega_5+\omega_3-\omega_1$ $\omega_4+\omega_4-\omega_1$	$\omega_5+\omega_5-\omega_2$ $\omega_5+\omega_4-\omega_1$	$\omega_5+\omega_5-\omega_1$

Table 1 – 5 tone third order mixing products.

Considering, for example, the spectral regrowth tone identified as ω_c we can see that the output signal depends on:

$$H_3(\omega_5,\omega_5,-\omega_2)=K_3-K_2\left[2H_2(\omega_5,-\omega_2)+H_2(\omega_5,\omega_5)\right] \quad (2)$$

and

$$H_3(\omega_5,\omega_4,-\omega_1)=K_3-K_2\left[H_2(\omega_5,-\omega_1)+H_2(\omega_4,-\omega_1)+H_2(\omega_5,\omega_4)\right] \quad (3)$$

If the multi-sine excitation could be considered narrow-band, i.e., if the channel's bandwidth is greater than the signal's bandwidth, then K_3 would be approximately constant. The mixing product arising from $H_2(\omega_x,\omega_y)$, where $\omega_x+\omega_y$ is at the 2[nd] harmonic, could also be considered constant since the relative bandwidth change with the tone spacing is very small.

Eq. (2) shows that this case is similar to the response of the system for a two-tone signal with tone spacing ω_5-ω_2.

Considering the previous analysis, we must be aware that the output signal at ω_c is the summation of the two terms in power and not in voltage, so the characterization of those terms is quite complex. Furthermore, from Table 1 we can state that all the mixing terms have at much two terms of the form $H_2(\omega_x,-\omega_y)$. So, if we manage to characterize each of those terms, individually, we could, in principle, get all the long term memory effects that we need for a multi-sine excitation.

III. Nonlinear Distortion Component Identification

In order to clearly identify each of those components, a two-tone test is performed and the result is computed according to Eq. (1). So, since the most important terms are the ones that vary with tone spacing, we start by first identifying the constant part of the expression. That is done from the asymptotic behavior of $Y(2\omega_2-\omega_1)$ at very low frequency separations, ideally zero Hz. So the two-tone contribution becomes:

$$Y(2\omega_2-\omega_1)=\left[K-2F_2(\omega_2,-\omega_1)\right]3X(\omega_2)X(\omega_2)X(-\omega_1) \quad (4)$$

where K is:

$$K=k_3-k_2H_2(\omega_x,\omega_x)-2k_2H_2(\omega_x,-\omega_x) \quad (5)$$

and $F_2(\omega_2,-\omega_1)$ is the term that varies with tone spacing.

This way, by changing the tone spacing, the different components can be obtained. If the tone spacing is made sufficiently small, it could be assumed a smooth frequency response so that K can also be extracted. But, since the terms $F_2(\omega_x,-\omega_y)$ in the multi-sine case, add in voltage, we must have them characterized both in amplitude and phase.

So, for each frequency component we need to solve the following equation:

$$H_3(\omega_x,\omega_x,-\omega_y)=K-2F_2(\omega_x,-\omega_y) \quad (6)$$

The computation of the system nonlinear transfer functions is achieved by using *Higher Order Statistics, HOS,* considering a two-tone as the input test signal [7]. This way, we can obtain the transfer function both in amplitude and phase.

These equations should be calculated for each tone spacing, which means that, if we have a n tone excitation, then we will have n-1 different tone spacing's involved, and so at least a linear system of n equations should be solved, corresponding to n-1 tone spacings and the constant, K. This system of equations is built by measuring the *HOS* for each two-tone signal at every different tone spacing. In a test with arbitrary number of tones, the system to be solved can be represented in a matrix form as:

$$\begin{bmatrix}1 & 0 & .. & 0\\ 1 & 2 & .. & 0\\ .. & .. & .. & ..\\ 1 & 0 & .. & 2\end{bmatrix}\begin{bmatrix}K\\ F_2(\Delta\omega)\\ ..\\ F_2\left[(n-1)\Delta\omega\right]\end{bmatrix}=\begin{bmatrix}H_3^{k}(.)/3\\ H_3^{df}(.)/3\\ ..\\ H_3^{(n-1)df}(.)/3\end{bmatrix} \quad (7)$$

Where: $H_3^{(n-1)df}(.)$ is the *Third Order Statistics* for a $(n-1)\Delta\omega$ tone spacing,

$F_2\left[(n-1)\Delta\omega\right]$ is the second order transfer function for a $(n\text{-}1)\Delta\omega$ tone spacing, and

n-1 is the number of tone spacings considered.

IV. Multi-sine Evaluation

In order to verify these assumptions, we have implemented a third order dynamic nonlinearity, presenting long term memory effects which were then used as our test system.

In the first case studied, the base-band equivalent filter was a low pass filter with 40 kHz cutoff frequency. Fig. 3 presents the amplitude response of that filter.

Next, the system was tested with a two tone excitation. Then, the different values of $F_2(\Delta\omega)$ were calculated using (7), and the output results computed according to (2-6) and Table 1.

Fig. 4 presents a comparison between the results obtained from the nonlinear model and the ones simulated with the new compact expressions, for a five tone uncorrelated input signal excitation.

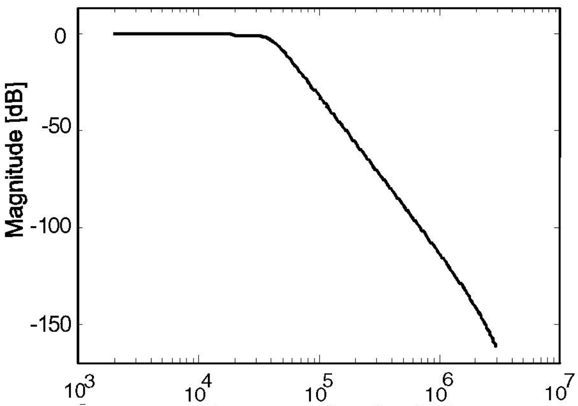

Fig. 3 – Magnitude response of the base-band low pass filter's transfer function.

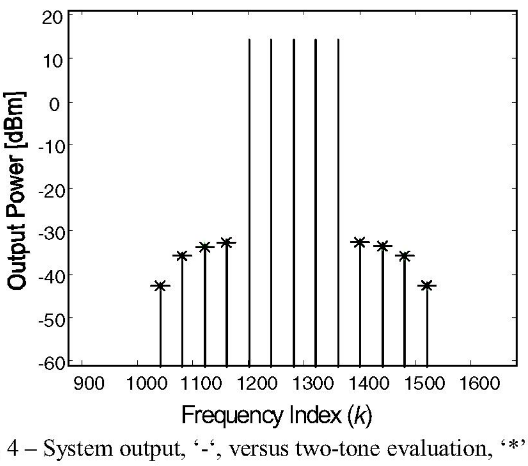

Fig. 4 – System output, '-', versus two-tone evaluation, '*'.

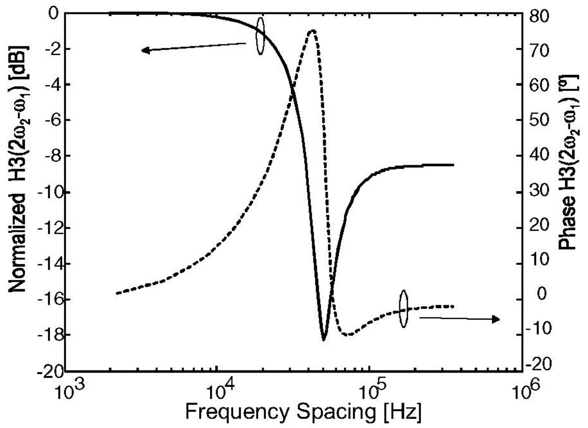

Fig. 5 – Normalized magnitude and phase of $H_3(2\omega_2,-\omega_1)$ for a base-band low pass filter of 40 kHz cutoff frequency.

Fig. 5 presents the $H_3(.)$ transfer characteristics [8]. As can be seen, below 40 kHz the base-band component and the constant component "*K*" add together. Above 40kHz the response tends to the constant value "*K*", since the terms $F_2(\Delta\omega)$ vanish as a consequence of the base-band filter shape.

The obtained error level is presented in Table 2, which states a perfect match between the model and the computed values.

Spectral Regrowth tone	ω_a	ω_b	ω_c	ω_d
Error [dB]	0.02	0.18	0.28	0.00

Table 2 – Error between proposed model and simulated values.

The same procedure was then applied to a different base-band shape filter, a band pass filter with 120 and 180 kHz lower and upper cutoff frequencies, Fig. 6.

Fig. 7 presents the model output and the computed values. The spectral regrowth presents now a distinct shape from the previous one. The magnitude response clearly reflects the band pass response imposed by the base-band.

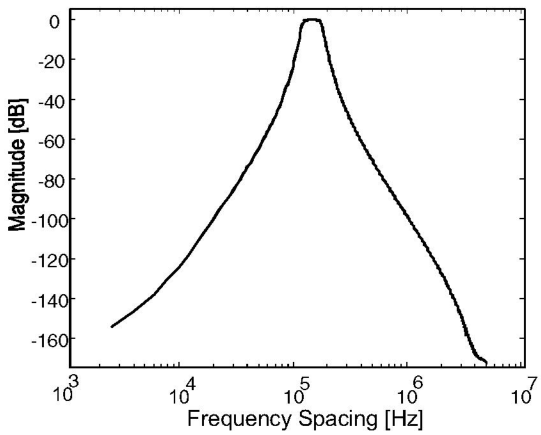

Fig. 6 – Magnitude response of the base-band band pass filter's transfer function.

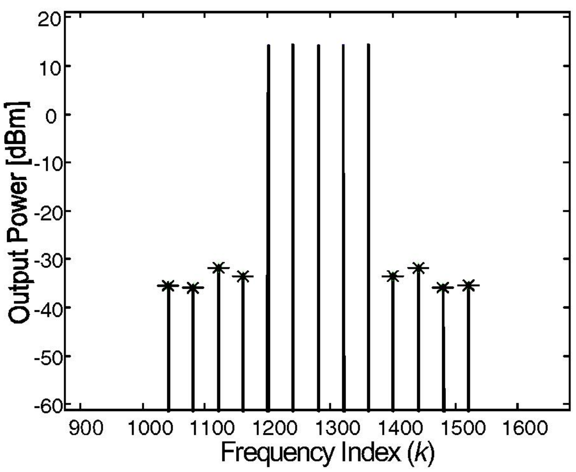

Fig. 7 – System output, '-', versus two-tone evaluation, '*'.

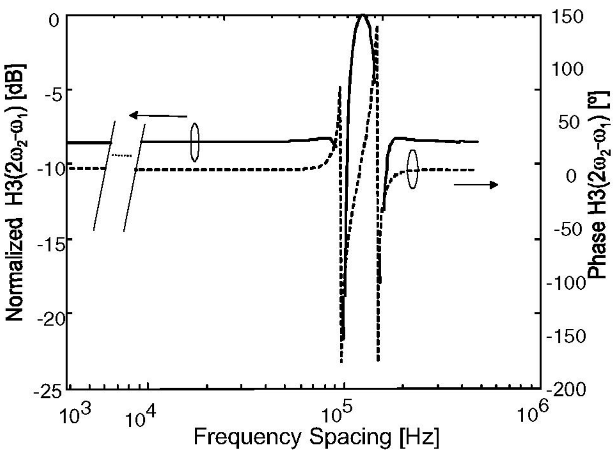

Fig. 8 – Normalized magnitude and phase of $H_3(2\omega2,-\omega1)$ for a pass band response of the base-band filter.

As see in Fig. 6 and Fig. 8, the filter shape is again somehow reflected in $H_3(.)$ through the base-band to RF up conversion process, which states that a careful study of the two tone IMD characteristics versus tone spacing, can indeed give valid information for predicting the multi-sine spectral regrowth.

V. CONCLUSION

The work now presented in this paper is a step forward in understanding the memory effect generation mechanisms. It allows the extrapolation of usual IMD test results to the prediction of the dynamic system's output to much more complex multi-sine excitations.

It is proved that the intermodulation distortion output of a third order dynamic nonlinearity can be fully described by a two-tone test, when the in-band and second harmonic frequency responses are flat. The computational framework developed is in perfect agreement with the system results, which states the validity of the method. This method also allows the prediction of the IMD shape based on the base-band response enabling this way a more efficient PA design.

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