

A Formal Procedure for Microwave Power Amplifier Behavioral Modeling

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Abstract – This work presents a formal behavioral modeling procedure for microwave power amplifier devices. By relying on the solid background of nonlinear system identification theory, a new behavioral model formulation was especially conceived to simplify the parameter set extraction from measurements made in a microwave laboratory. Indeed, the model now proposed is an optimal approximator of the real PA, for the nonlinear order considered, and is extracted, in a separable way, from measurements of the PA response when the device is excited by one of the easiest signals to generate in a RF laboratory: a multi-sine.

Index-Terms – Behavioral model, power amplifiers, measurement.

I. INTRODUCTION

As was shown in a recent review publication [1], behavioral modeling of wireless PAs has become a research area whose interest has been steadily increasing. Nevertheless, that work also shows that despite different new models are continuously being proposed, most of the approaches lack in a formal justification and support. The situation is even weirder because, although the representations are black-box in nature, and so completely relying on observations of input-output behavior, the information disclosed on the model extraction procedure is usually very scarce.

The consequence of this is that the models this way advanced have no guaranteed predictive capabilities. This means that, although the model is usually shown to perform well for the device it was extracted, and within the vicinity of the extraction point (i.e. waveform and mean power of the stimulus used to get the extraction's data) many times no one can tell anything about the capability of the model to extrapolate beyond these observations. That is, nothing is said on what happens if that modeling methodology is applied to a different PA technology (or simply a distinct bias point), or even if one wants to predict the response of the system to the same stimulus class, but with a different input power or waveform.

This paper presents a model methodology in which all these issues are thoroughly addressed. In fact, as will be shown in the next sections, the model format is guaranteed to be general (although approximate in nature), and was conceived to be extracted from the measurements already available in a microwave laboratory.

II. THE ADOPTED BEHAVIORAL MODELING METHODOLOGY

This section is devoted to explain the underlying ideas of the adopted behavioral modeling methodology.

To achieve guaranteed generality and predictability, we based our approach in the groundwork of nonlinear system identification theory. This says that any single-input single-output forced nonlinear dynamic system that is stable and of fading memory (a good theoretical framework for many microwave circuits as power amplifiers and mixers) can be represented by a cascade of a one-to-many linear system with memory, followed by a many-to-one nonlinear memoryless system [2,3]. So, one of its most common implementations is the nonlinear finite impulse response, FIR, filter

$$y(s) = f_{NL}[x(s), x(s-1), \dots, x(s-M)] \quad (1)$$

in which the 1-to-($M+1$) linear dynamic system was implemented as a ladder of M unit delays, and the nonlinear function, $f_{NL}[\cdot]$, can be any $(1+M)$ -to-1 universal approximator, $R^{(M+1)} \rightarrow R$. Two possible implementations are known (and widely used) for this static nonlinear map. They are the artificial neural network, ANN, – leading to the so called time-delay neural network, TDNN – and the multi-dimensional polynomial – which leads to the general polynomial filters [4].

Choosing between these two different possibilities is not easy because they both present comparative advantages and drawbacks [1]. However, since, in this case, we were interested in supporting our modeling approach with a systematic parameter extraction methodology, we adopted the polynomial filter. Being a nonlinear representation that is linear in its parameters, it allows a direct extraction, e.g. following the least mean square error, LMS, technique. So, the adopted model format has the form of:

$$y(s) = \sum_{n=1}^N \left[\sum_{m_1=0}^M \dots \sum_{m_n=0}^M h_n(m_1, \dots, m_n) x(s-m_1) \dots x(s-m_n) \right] \quad (2)$$

or

$$y(s) = \sum_{n, \mathbf{m}} h_{n, \mathbf{m}} \psi_{n, \mathbf{m}}[x(s), \dots, x(s-M)] \quad (3)$$

in which the basis functions of the model, $\Psi_n[x(t)]$, are monomials or combinations of them, i.e., polynomials and \mathbf{m} is the vector of time delays.

Although the extraction of the parameter set of this type of model – the nonlinear impulse responses $h_n(m_1, \dots, m_n)$ – can, in theory, be carried out via the solution of a linear system of equations (for example derived from a LMS identification algorithm), this is not that easy in practice, because the model involves a very large number of parameters [5,6] and the system tends to be ill-conditioned [4]. The way usually found for circumventing this difficulty is to rely on some optimization process, in which one widely used method is based on extracting the model when it is inserted in an adaptation loop. Unfortunately, since this has the disadvantage of destroying the direct extraction provided by the polynomial filter format, we decided to solve it in a completely different way [7].

To understand how this was accomplished, we must recall that the LMS system becomes ill-conditioned when the input-output observations, taken over a sample window, lead to a group of matrix lines which are almost linearly dependent. This might be due to two different reasons.

One is due to the fact that the selected stimulus may not be sufficiently rich to ensure the excitation of all important (up to the desired nonlinear order and memory span) device under test, DUT, states. And so, if these are not excited, they do not produce any response, and we become impeded of accessing the DUT's properties. For the purpose of illustration, one known example in linear system identification is the use of a single frequency sinusoidal stimulus versus the use of an ideal impulse.

The other reason for the presence of this ill-conditioned system derives from the fact that the model may have many coefficients contributing to the same output. Again, for example, in case of linear systems, this corresponds to having many of the coefficients of the FIR filter contributing to the response at the same time instant, for a non particular input. Whereas, when the excitation is an impulse, the relations of the FIR coefficients and the time observations become automatically decoupled. Such a model is said to be orthogonal for that particular input.

These problems, widely known from linear system identification, are aggravated in the case of nonlinear models. So, if the systematic extraction procedure is to be kept, some alternative solution should be investigated.

The underlying idea of this work was to (i) use, as the excitation, a signal known to be sufficiently rich and (ii) find a polynomial filter format known to be orthogonal to this stimulus, i.e., in which there is a one-to-one relation between some set of properties of the response to the excitation used in the extraction, and the model parameters. Note that such a solution is indeed the best in guaranteeing a well-conditioned extraction system of equations as it leads to a diagonal matrix.

III. NONLINEAR DYNAMIC MODEL IDENTIFICATION

According to the previous Section, the model formulation should be the one of (2). However, in the extraction phase, its kernels must be rewritten in an equivalent way, so that this auxiliary formulation becomes orthogonal to the excitation used in the parameter set extraction. In addition, this excitation should be a-priori known to be capable of exciting all DUT's nonlinear dynamic states up to order N and memory span M .

Back in the forties and fifties, Wiener demonstrated that the white noise was such a signal [2]. Unfortunately, this type of signal is not appropriate for microwave devices as it is difficult to be generated and processed in a RF laboratory. In fact, nowadays, microwave instrumentation is devoted to generate sinusoids or a set of sinusoids, the multi-sine. Fortunately, it has been shown that a large set of realizations of a random multi-sine (a set of equally spaced tones of constant amplitude but with randomized phases) produces the same output in a dynamic nonlinear system of fading memory as the white Gaussian noise [8]. Hence, this was the excitation adopted.

The next step consists in building of set of basis functions $\Psi_1[x(t)]$, ..., $\Psi_N[x(t)]$ that is orthogonal to this type of input, i.e., so that

$$\langle \psi_{n_1}[x(t)] | \psi_{n_2}[x(t)] \rangle \neq 0 \quad \text{if and only if} \quad n_1 = n_2 \quad (4)$$

where the average used in the definition of this inner product is made across all multi-sine realizations.

For that, it was assumed that the model's 1-to-($M+1$) input linear dynamic system was such that the input $x(t)$ would be decomposed into a set of $K=(M+1)/2$ in-phase and quadrature components:

$$x_k^I(t) = A_k \cos(\omega_k t + \phi_k), \quad x_k^Q(t) = A_k \sin(\omega_k t + \phi_k) \quad (5)$$

Using the orthogonality of sine functions of different frequencies, (4) demands that each $\Psi_n[x(t)]$ produces components at a single frequency. Unfortunately, that is not the case for the monomials of (2). For example, the cubic monomial excited by a cosine of amplitude A and frequency ω produces an output at the third harmonic, of amplitude $(1/4)A^3$ but also another one at the fundamental of amplitude $(3/4)A^3$. However, the linear combination of a monomial of 3rd order with another one of 1st order, $[4x(t)^3 - 3A_0^2x(t)]$, already produces a polynomial (known as the 3rd order Chebyshev polynomial) that is orthogonal for a cosine excitation of amplitude A_0 (thus, the amplitude with which the model is to be extracted).

This concept was generalized to the multi-dimensional case, leading to the following in-phase, $\Psi_n^I(\omega_1 + \dots + \omega_n, s)$, and quadrature, $\Psi_n^Q(\omega_1 + \dots + \omega_n, s)$, basis set:

$$\begin{aligned}
& \Psi_n^{I,Q}(\omega_1 + \dots + \omega_n, s) \\
&= \Psi_{(n-1)}^I(\omega_1 + \dots + \omega_{n-1}, s) \Psi_1^{I,Q}(\omega_n, s) \\
&\quad - \Psi_{n-1}^Q(\omega_1 + \dots + \omega_{n-1}, s) \Psi_1^{Q,I}(\omega_n, s) \\
&\quad - A_0^2 \sum_{u=1}^{n-1} \sum_{k_1=1}^{n-1} \dots \sum_{k_u=k_{u-1}+1}^{n-1} \delta(\omega_{k_1} + \omega_n) \dots \delta(\omega_{k_u} + \omega_n) \quad (6) \\
&\quad \cdot \prod_{v_1=1}^{n-1} \dots \prod_{v_u=v_{u-1}+1}^{n-1} [1 - \delta(\omega_{v_1} - \omega_k) \dots \delta(\omega_{v_u} - \omega_k)] \\
&\quad \cdot \Psi_{n-2}^{I,Q}(\omega_1 + \dots + \omega_{k-1} + \omega_{k+1} + \dots + \omega_{n-1}, s)
\end{aligned}$$

Now we have an auxiliary model of the form of (3) whose coefficients use the general form of $C_n^{I,Q}(\omega_1, \dots, \omega_n)$ and whose basis functions are orthogonal to a random multi-sine. So, its parameter extraction process can be carried out testing the DUT with a series of R realizations of the considered multi-sine of randomized phases (in which the number of tones, K , is selected to meet the Nyquist sampling criterion), recording R windows of input-output data, and computing the following ratios between the input-output higher order cross-correlations and the input auto-correlations [9]:

$$\begin{aligned}
& C_n^{I,Q}(\omega_1, \dots, \omega_n) \\
&= \frac{\frac{1}{R} \sum_{r=1}^R \left[\sum_{s=1}^S y(s) \Psi_n^{I,Q},r(\omega_1 + \dots + \omega_n, s) \right]}{\frac{1}{R} \sum_{r=1}^R \left[\sum_{s=1}^S \Psi_n^{I,Q},r(\omega_1 + \dots + \omega_n, s) \cdot \Psi_n^{I,Q},r(\omega_1 + \dots + \omega_n, s) \right]} \quad (7)
\end{aligned}$$

In fact, this expression evaluates the magnitude of the $y(t)$ components, or projection, on each of the expansion basis.

At this point, two different situations must be addressed. One refers to some exceptions to the model's orthogonality. These are expansion terms that, e.g., in third order, produce output components of the form $\omega_i + \omega_j - \omega_l$. As they are correlated with the first order component at ω_i , they can not be separated from it using correlation. There are, however, two ways of solving this exception. One is based on the fact that these are the limit of $\omega_i + \omega_j - \omega_l$ when ω_l tends to ω_j . So, as suggested in [2] for a similar problem of the Wiener G functionals, these can be identified by assuming continuity of the model kernels. The other alternative relies on repeating the test of the DUT at a different power level, realizing that the amplitude of the third term varies cubically with the input level, while the first order term varies linearly. This latter strategy was the one adopted in the present implementation.

The second exception occurs because we are using evenly spaced multi-sines, where there are many mixing products of the form $\omega_j + \omega_k - \omega_l$ that fall at the same output frequency. However, since every multi-sine realization, r , uses a new set of independent and randomized phases, $\phi_k(r)$, and as these $\omega_i + \omega_j - \omega_l$ products have different resultant phases $[\phi_i(r) + \phi_j(r) - \phi_l(r)]$,

they will be easily separated via correlation.

At this stage, we reached the desired auxiliary nonlinear model, which is orthogonal to the Fourier expansion of the input, and thus allowed a laboratory extraction procedure in a systematic and direct way. However, since its basis functions are simply linear combinations of monomials, it can be re-written in a non-orthogonal way as

$$y(s) = \sum_{n=1}^N \left[\sum_{k_1=1}^K \dots \sum_{k_n=1}^K H_n(\omega_{k_1}, \dots, \omega_{k_n}) x_{k_1}^I(s) \dots x_{k_n}^Q(s) \right] \quad (8)$$

Now, notice that, if the $H_n(\omega_1, \dots, \omega_K)$ represent our system when the input is described in the frequency-domain of Eq. (5), then they are nothing but the nonlinear transfer functions (in the Volterra series sense) of our DUT. Therefore, the coefficients, $h_n(m_1, \dots, m_n)$, of the desired time-domain model of (2) can be finally obtained by a straightforward multi-dimensional inverse Fourier transform.

This completes the nonlinear dynamic model identification process.

IV. ILLUSTRATIVE MICROWAVE PA BEHAVIORAL MODELING RESULTS

For the purpose of illustration, we picked up a typical power amplifier design, described by its nonlinear equivalent circuit in a commercial microwave CAD environment. Assuming this equivalent circuit model as the "true" PA, we then extracted the behavioral model of (2) using the methodology just explained.

The PA was designed to handle a WCDMA signal with a carrier of 1.9GHz. In order to show the model capabilities for dealing with nonlinear dynamic systems, the output bias network was designed so that it could deliberately introduce a reasonable amount of nonlinear memory.

Fig. 1 reports a two-tone AM/AM plot - for a separation of 2MHz - where the dynamic and nonlinear effects are shown.

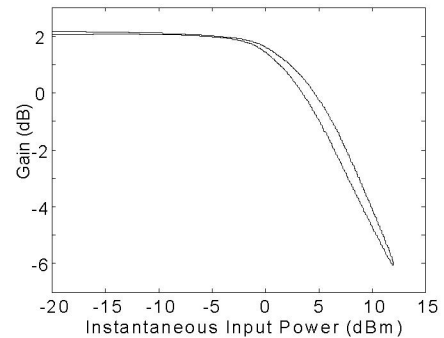


Fig. 1. Instantaneous AM/AM amplifier characterization.

For this PA, a fifth order lowpass equivalent model was extracted using a 7-tone multi-sine around an input excitation of -2dBm. This spans the instantaneous input power up to

6.5dBm (clearly in a zone where gain compression and long term memory are both evident).

As the adopted model formulation is completely general, this example of 7-tones, leading thus to a model of 7 delays, up to 5th order, involved the extraction of 1106 coefficients.

Fig. 2 shows a comparison between observed and predicted responses to a 3G-WCDMA signal driving the amplifier with an average power equal to the one of the random multi-sine used for model extraction.

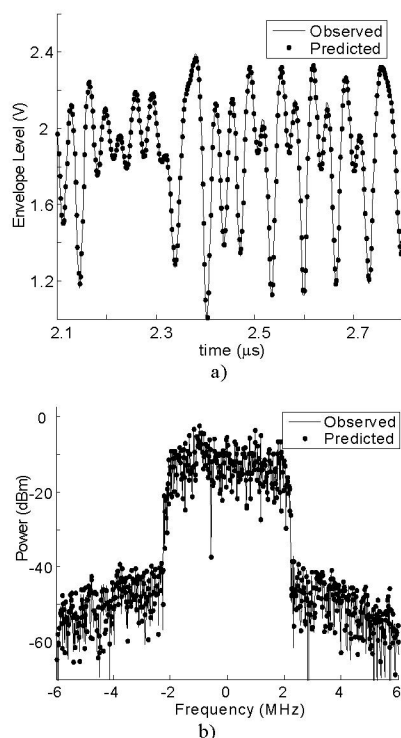


Fig. 2. Comparison between observed and predicted responses to a 3G-WCDMA signal with an average power of -2dBm. a) Time domain envelope amplitude. b) Frequency domain spectrum.

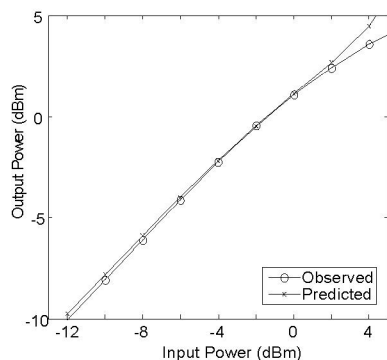


Fig. 3. Observed and predicted output average power measured with a 3G-WCDMA signal.

Fig. 3 shows the predicted and observed average output powers when the input signal level is swept above and below the power of the extraction point. As expected from the local behavior of the polynomial approximation, the error increases

above and below the extraction level. Nevertheless, it should be noticed that there is still a reasonable wide range of input powers where the model gives useful results.

In fact, the presented results prove the robustness of the proposed modeling approach, as the signal format used for model extraction was clearly different from the one used for model validation.

V. CONCLUSIONS

Using the formal groundwork of nonlinear system identification theory, we derived a model that is guaranteed to be general [up to the memory span $(1+M)$ and nonlinear order, N , considered], predictive, and amenable for systematic extraction in a microwave laboratory.

Not only this modeling strategy represents an important breakthrough in the existing microwave behavioral modeling scenario, as it is also believed to help other modeling alternatives, using, for example, ANNs, which are known to present advantages over the polynomial filters, but are much more difficult to extract.

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