Designing Band-Pass Multisine Excitations for Microwave Behavioral Model Identification

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Abstract — **This work deals with the design of band-pass multisine excitations appropriate for microwave nonlinear model extraction and validation. Several examples show that the probability with which each instantaneous amplitude value is reached plays a determinant role on the response of a memoryless nonlinear system. Moreover, further tests made with nonlinear dynamic systems have shown that even the signal probability density function is insufficient, and one needs to know the signal's waveform, or the complete multisine amplitude and phase distribution.**

Index-Terms – Nonlinear systems, Behavioral science, Waveform Analysis

I. INTRODUCTION

Recently, there has been an increasing interest in behavioral models of nonlinear microwave and wireless systems because they can reduce the amount of the circuit's state variables, enabling, therefore, the efficient simulation of disparate circuit types and very large systems. However, since these nonlinear behavioral models have to rely on input-output observations, their accuracy and predictive capabilities are highly conditioned by the data used to extract their parameter set. As a matter of fact, since there are already a group of model structures recognized for their universal approximation abilities [1], [2], the behavioral model problem resumes to a careful selection of input-output data (the so-called training data for artificial neural networks). Hence, behavioral model quality is ultimately determined by the stimulus used for its parameter set extraction.

Under the group of possible stimulus, first Wiener, and then Lee and Schetzen [3] have proved that the data collected using a white Gaussian noise stimulus is enough to extract a full dynamic model of any nonlinear dynamic operator of fading memory.

From then, various authors have tried to substitute this random excitation by periodic (and, in the microwave and wireless fields, band-limited) signals as these enable the use of powerful tools of digital signal processing [4]-[7]. As a particular example of this large class of periodic signals Schoukens and Pintelon [8] have shown that the response of the system converges to the same result when it is excited by either white Gaussian noise, a multisine of random amplitude and phases (named periodic noise), or even a multisine of constant amplitude and random phases (named random multisine). Therefore, a necessary conclusion that can be drawn from their work is that a multisine could also allow the extraction of a full nonlinear dynamic behavioral model. This provides such signal class with an enormous relevance in the field of behavioral modeling extraction and validation.

This paper gives a contribution to the problem of designing band-pass multisines for microwave nonlinear model extraction and validation paying particular attention to the statistical characteristics (probability density function) of the synthesized signal. In fact, using an algorithm conceived to generate a multisine with specified power spectral and probability density functions, it shows that, more important than the range of instantaneous amplitude values covered by the multisine, is the probability with which each value is reached.

But, then, it also shows that even this statistical measure is insufficient to fully predict the impact of such a multisine signal in a nonlinear system with memory.

II. THE ROLE OF EXCITATION'S *PDF* ON NONLINEAR MEMORYLESS SYSTEM'S RESPONSE

Nonlinear memoryless systems react instantaneously to the excitation's value. So, we can anticipate that their instantaneous output must be completely determined by the domain of stimulus amplitudes. However, recognizing that most of the system's output metrics, like output power, power spectral density, PSD, or adjacent channel power ratio, ACPR, share a statistical average nature, we are led to the conclusion that, as important as the range of amplitude values covered by the output, should be the probability with which they are reached.

Such conclusion comes intuitively to our minds if we recall the physical interpretation of the probability density function, *pdf*, of a certain finite range of signal values as the percentage of time the signal passes through that range. Then we must also recognize that, in average, what matters is not the instantaneous amplitude itself, but the value weighted by the *pdf*. So, although a certain very high instantaneous amplitude can determine the signal's amplitude span, it will actually become almost irrelevant to the system's output if its associated *pdf* is very low.

This was what led previous authors [9] to abandon the peak-to-average ratio as a faithful predictor of output system distortion. It also led manufacturers of arbitrary waveform generators to give information on the probability distribution function, or, more commonly, the complementary cumulative distribution function.

To illustrate these qualitative conclusions we selected several signals of zero mean (zero dc offset) and equal power, but with notoriously distinct *pdf*.

Figure Fig. 1 presents the $pdf_x(x)$ of three signals commonly used for memoryless (and, sometimes, also dynamic) model extraction and validation.

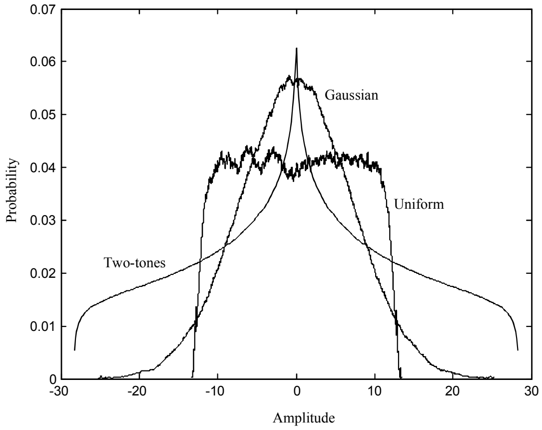


Fig. 1. Probability density function, $pdf_x(x)$ of a two-tone, and two multisines of uniform and Gaussian distribution, all with the same integrated power.

The first one is an equal amplitude of 7V two-tone signal. The second and third signals are multisines of uniform and Gaussian *pdf*, respectively. They correspond to test the system with uniformly or Gaussian distributed band-limited white noisy waveforms of total power equal to the two-tone. These multisines were designed using a modified version of the Schouckens algorithm [10] so that their *pdf* approximates the *pdf* of previously synthesized noise sequences.

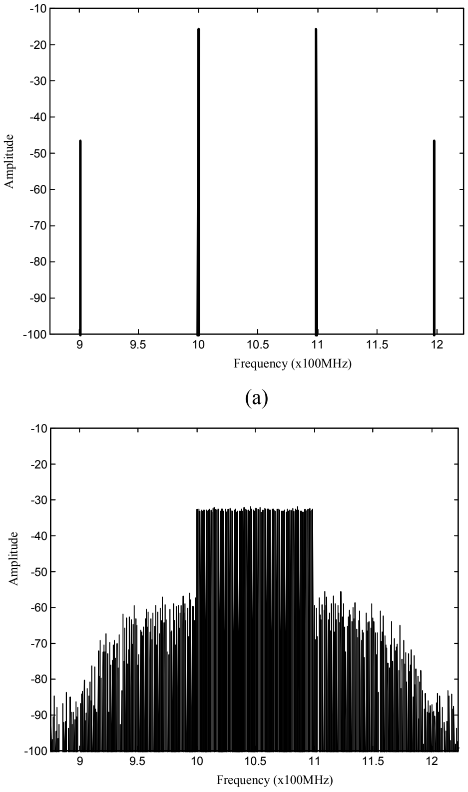
An outline of the algorithm is:

1 – Synthesize a noisy signal with the specified *pdf* and re-order its instantaneous amplitude values in descending order. This creates the vector of *pdf* bins for the noise.
2 – Synthesize an equal amplitude multisine with the prescribed number and frequency position of tones.
3 – Re-order its instantaneous amplitude values in descending order, recording the time samples where they stood. This creates the vector of *pdf* bins for the multisine.
4 – Substitute the amplitudes of the multisine vector of *pdf* bins by the one of the noise.
5 – Restore these amplitudes in the original time samples of the multisine, creating a new multisine with the desired *pdf*.
6 – Calculate the DFT of this signal, and level-off the resulting tone-amplitudes, so that the total power is kept, maintaining the obtained phases. This is the desired multisine we sought.
7 – If the process of tone amplitude level-off has modified the multisine *pdf* to an unacceptable error, repeat the algorithm, using as the starting multisine the one this way synthesized, until an acceptable error is reached.

Fig. 2 shows the first zone output obtained by passing those signals through a nonlinear memoryless system defined by:

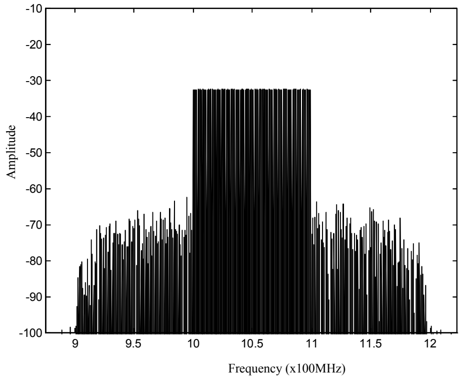
$$y(t) = \tanh\left(\frac{x(t)}{20}\right) \tag{1}$$

This memoryless nonlinearity was selected because it shows a linear zone followed by smooth saturation. It is, thus, representative of many nonlinearities found in practice, as the ones encountered in power amplifiers.



(a)

(b)



(c)

Fig. 2. Output power spectrum of the memoryless system of (1) excited by equal power two-tone signal, ACPR=31dB - (a); uniformly distributed, ACPR=38dB - (b) and Gaussian distributed multisines, ACPR=31dB - (c).

ACPR values of 38dB and 31dB for the multisine of uniform and Gaussian *pdf*, respectively, should be compared to an IMR of 31dB (extrapolated ACPR for signals of discrete power spectra).

III. THE ROLE OF EXCITATION *PDF* ON NONLINEAR DYNAMIC SYSTEM'S RESPONSE

System's dynamics add more dimensions to the problem of excitation design because the output will no longer be determined by the present input, but by all past inputs within a certain memory span. For example, in the field of polynomial approximation theory, the system's output (in a discrete time environment $t \rightarrow sT_s$, where T_s is the sampling period) will cease to be described by a simple power series to be represented by the following dynamic polynomial [1]:

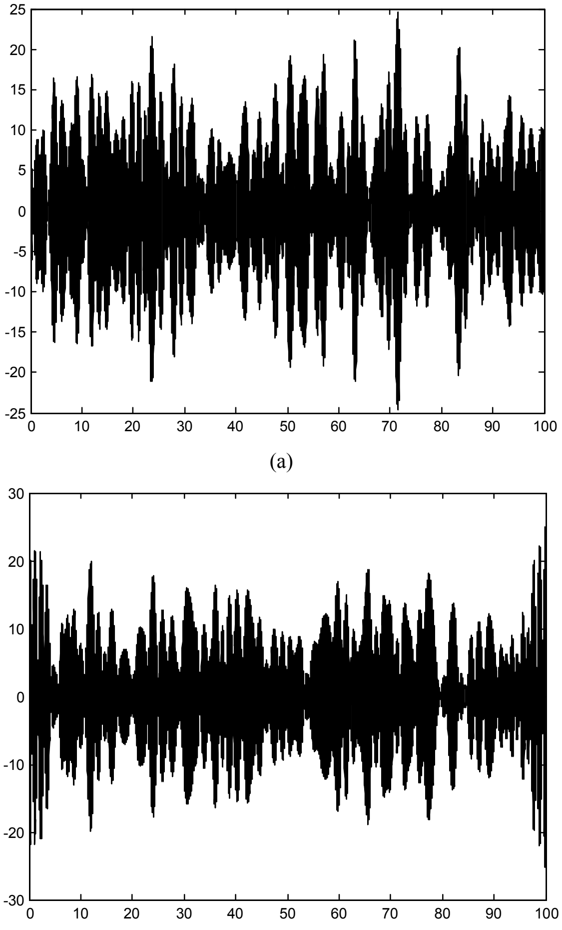
$$\begin{aligned} y(s) &\approx P[x(s),x(s-1),\ldots,x(s-Q+1)] \\ &= \sum_{n=1}^{N}\sum_{q_1=0}^{Q-1}\cdots\sum_{q_n=0}^{Q-1} h_n(q_1,\ldots,q_n)x(s-q_1)\ldots x(s-q_n) \end{aligned} \quad (2)$$

which shows that the output will now be determined by the joint statistics of the present input, $x(s)$, and its delayed versions, $x(s\text{-}1),\ldots, x(s\text{-}q),\ldots, x(s\text{-}Q)$.

This means that, since the memoryless system's output, $y(s)$, is totally defined by $x(s)$, any multisine having the same power spectrum and *pdf* would produce the same output; whereas, in a dynamic system, the complete waveform must be specified. So, for a nonlinear dynamic system, each multisine appears to be unique, and we can no longer build it by simply imposing the power spectrum (number, position and constant amplitude of the tones) as before. In the same way the dynamic system will be sensitive to the actual signal's time-domain waveform, in the frequency-domain the multisine complete amplitude and phase spectrum must be specified.

This constitutes a rather intuitive conclusion if we recall the central limit theorem which predicts that if $x(s)$, $x(s\text{-}1),\ldots,x(s\text{-}Q)$ are statistically independent (approximately white power spectral density function within the system's bandwidth) their passage through a linear dynamic filter converts any $pdf_x(x)$ into a Gaussian distribution. Therefore, as was previously shown by the authors [7], the output ACPR of a linear filter-memoryless nonlinearity connection (the so-called Wiener model) can be very similar, even though it results from two multisines of very distinct *pdf*.

Another illustrative example is described by figures Fig. 3 and Fig. 4. Two multisines of 100 tones of equal amplitude, evenly located within the bandwidth [1GHz, 1.1GHz], were synthesized by approximating a Gaussian *pdf* of zero mean and 7V standard deviation, but imposing no more restrictions to the phases of each tone. This led to multisines whose periodic envelope waveform is clearly distinct, as is depicted in Fig. 3.



(a)

(b)

Fig. 3. Two periods of the time-domain envelope waveforms of the two multisines of equal Gaussian *pdf* and power spectrum.

Although the response of our memoryless nonlinear system produced equal ACPR values of 28dB, Fig. 4 shows that the stochastic distortion power distribution in the first zone output is remarkably distinct. This is a direct consequence of the fact that, since each of these spectral regrowth lines is the vectorial addition of many intermodulation products of the form $\omega_x=\omega_1+\ldots+\omega_{n+1}-\omega_{n+2}-\ldots-\omega_{2n+1}$, whose phases are $\phi_x=\phi_1+\ldots+\phi_{n+1}-\phi_{n+2}-\ldots-\phi_{2n+1}$, their amplitude becomes a function of the actual distribution of phases among the original tones.

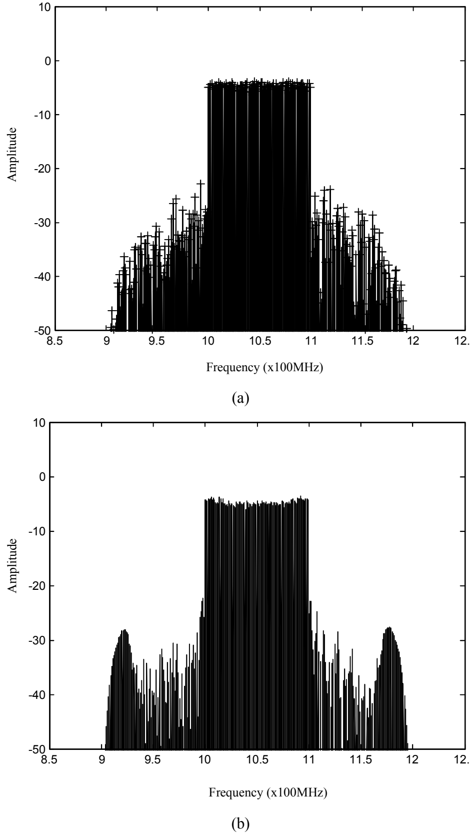


Fig. 4. Output power spectrum of the two multisiness after being processed by our memoryless nonlinearity. Note that even though the integrated ACPR is 28dB for both signals, their distribution of power within the spectral regrowth is completely distinct.

Naturally, if now these two distinct first zone outputs were passed through a linear narrowband band-pass filter (whose connection after the memoryless nonlinearity constitutes a dynamic nonlinearity usually known as the Hammerstein model) the resulting ACPRs would become dramatically different. Indeed, the output power spectrum of the tested Hammerstein model, revealed ACPR values of 28dB and 32dB for the narrower (100MHz equivalent noise bandwidth) and 29dB and 28dB broader (920MHz equivalent noise bandwidth) linear filters.

IV. CONCLUSIONS

This paper dealt with the design of band-pass multisine excitations for microwave nonlinear behavioral model extraction and validation, paying particular attention to the stimulus probability density function. It was shown that the excitation *pdf* plays a determinant role on the response of nonlinear memoryless systems, but that it was unfortunately insufficient for dynamic systems. In these latter cases, an accurate prediction of the system's output requires the complete knowledge of the time-domain waveform (at least within the system's memory span) or the complete, amplitude and phase, multisine spectrum.

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