Simulation of RF Circuits Driven By Modulated Signals Without Bandwidth Constraints

José Carlos Pedro and Nuno Borges Carvalho

e-mail: jcpedro@ieee.org and nborges@ieee.org
Instituto de Telecomunicações – Universidade de Aveiro, 3810 AVEIRO – Portugal

Abstract **— A new nonlinear analysis technique of RF circuits subject to modulated signals is discussed. As the envelope and carrier are uncorrelated, the circuit is modeled by a multi-rate partial differential equation. The carrier response is then simulated with harmonic-balance for each envelope time-step, without imposing any restrictions to signal bandwidth.**

I. Introduction

The nowadays-standard nonlinear RF and microwave circuit simulator tool is the harmonic-balance technique, HB. Its more usual implementation is based on a so-called mixed-mode algorithm that handles the linear sub-circuit in frequency-domain, and the nonlinear elements in time-domain. Then, a nodal current error equation is formulated in frequency-domain, and solved with a multi-dimensional Newton-Raphson iteration scheme. Because the nonlinear elements are treated in time-domain, HB must rely on a domain transformation tool, usually the DFT (or its fast algorithm the FFT). Unfortunately, this restricts HB applicability to signals that are amenable for a common DFT, or, at least, a N-dimensional DFT representation. So, although it is quite popular for simulating circuits driven by stereotype forcing functions as single-tone or two-tone, it has been difficult to apply to typical real telecommunication signals. This has opened a new field in RF and microwave circuit simulation tools capable of meeting these practical engineering needs.

It is widely accepted that excitations of modern wireless telecommunication circuits are efficiently represented by an RF or microwave carrier modulated by some kind of base-band information signal. When compared to the RF carrier, the envelope usually presents a spectrum that extends to significantly lower frequencies, but also of much higher spectral content. While the carrier can be easily described by a CW signal, the envelope requires a large number of closely separated tones, or, more realistically, a dense spectrum.

Such composite signals cannot be successfully approximated as periodic or quasi-periodic (2-periodic) excitations, falling outside the applicability range of HB. So, they demand the ancient time-step integration procedures of SPICE like programs. Unfortunately, time-step integration of an RF or microwave circuit ordinary differential equation, ODE, driven by an high frequency carrier can be, either inadequate to the format the RF engineer seeks, very time-consuming, or impossible at all, if the circuit includes dispersive distributed elements.

The Envelope Transient Harmonic Balance technique, ETHB, [1]-[3] is a true mixed-mode technique, precisely conceived to match these needs. Treating the envelope as a slowly varying signal modulating the circuit's behavior to the RF carrier, it converts the original ODE into another differential equation of time-varying Fourier coefficients. This frequency-domain representation of the carrier response, is then determined by HB in an envelope time-step by time-step basis. However, since the method assumes the envelope is a slowly varying modulating signal, it is automatically restricted to excitations that occupy only a small fraction of the linearized circuit's pass-band.

The main goal of this paper is to describe a new and general mixed-mode simulation tool that does not impose such excitation signal bandwidth restrictions.

II. Theoretical Formulation

Consider the following system of ordinary differential equations describing a general nonlinear circuit.

$$\mathbf{i}[\mathbf{y}(t)] + \frac{d\,\mathbf{q}[\mathbf{y}(t)]}{d\,t} = \mathbf{x}(t) \qquad (1)$$

$\mathbf{x}(t)$ and $\mathbf{y}(t)$ stand for the excitation and the state-variable vectors, respectively. $\mathbf{i}[\mathbf{y}(t)]$ represents memoryless linear or nonlinear elements, while $\mathbf{q}[\mathbf{y}(t)]$ models memoryless linear or nonlinear charges (capacitors) or fluxes (inductors).

In cases where $\mathbf{x}(t)$ can be assumed as a composite signal where a carrier is modulated by an uncorrelated, but in any other respect general (e.g., not necessarily real – $\mathbf{X}_m=\mathbf{X}_{-m}^*$ or $\mathbf{X}_m\neq\mathbf{X}_{-m}^*$), envelope:

$$\mathbf{x}(t)=\mathbf{X}_0+\left[\sum_{m=-\frac{Q-1}{2}}^{\frac{Q-1}{2}}\mathbf{X}_m\,\mathrm{e}^{j\Omega t}\right]\left(\mathrm{e}^{-j\omega_c t}+\mathrm{e}^{j\omega_c t}\right) \qquad (2)$$

such that the envelope time and frequency variables, $t_e \leftrightarrow \Omega$, can be considered independent from the correspondent carrier variables, $t_c \leftrightarrow \omega_c$, the nonlinear ODE of (1) can be converted into a multi-rate partial differential equation, MPDE, [4], [5] dependent on these new two time variables t_e and t_c:

$$\mathbf{i}[\mathbf{y}(t_e,t_c)]+\frac{\partial\,\mathbf{q}[\mathbf{y}(t_e,t_c)]}{\partial t_e}+\frac{\partial\,\mathbf{q}[\mathbf{y}(t_e,t_c)]}{\partial t_c}=\mathbf{x}(t_e,t_c) \qquad (3)$$

in a similar way as multi-dimensional Fourier transform, MDFT, based HB was introduced by Rizzoli *et al.* [6].

This MPDE can now either be solved in time-domain, using the traditional time-step integration procedure, or, in frequency-domain, by an appropriate HB algorithm. The MDFT-HB is a previous example where (3) was solved in frequency-domain for both t_e and t_c time-scales [4],[6], while [5] is an example where time-domain was used for both t_e and t_c. However, as was recently anticipated by Roychowdhury [7], this MPDE can be solved with any appropriate combination of time and frequency-domain techniques. Obviously, from what was said in the introduction, best efficiency is obtained, in the present case, if time-step integration is used for the envelope, and HB for the carrier. This results in the following system of (2K+1) transient envelope HB equations:

$$\mathbf{I}(t_e,k\omega_c)+\left.\frac{\partial\mathbf{Q}[\mathbf{Y}(t_e,\omega_c)]}{\partial t_e}\right|_{k\omega_c}+jk\omega_c\mathbf{Q}(t_e,k.\omega_c)=\mathbf{X}(t_e,k\omega_c) \qquad (4)$$

in which $\mathbf{I}(t_e, k.\omega_c)$, $\mathbf{Q}(t_e, k.\omega_c)$, $\mathbf{Y}(t_e, \omega_c)$ and $\mathbf{X}(t_e, k.\omega_c)$ are the t_e time varying Fourier components of the memoryless nonlinearities, the state variables and the excitation, respectively. The discretization of (4) using the backward Euler rule leads to the following system of difference equations in the above Fourier coefficients:

$$\begin{aligned}h_n.\mathbf{I}(t_{e_n},k\omega_c)+\mathbf{Q}[\mathbf{Y}(t_{e_n},\omega_c)]\big|_{k\omega_c}+h_n.jk\omega_c\,\mathbf{Q}(t_{e_n},k\omega_c)\\=h_n.\mathbf{X}(t_{e_n},k\omega_c)+\mathbf{Q}[\mathbf{Y}(t_{e_{n-1}},\omega_c)]\big|_{k\omega_c}\end{aligned} \qquad (5)$$

The proposed mixed-mode method operates by integrating (5) in a t_e time-step by time-step basis (h_n), starting from the initial conditions $\mathbf{X}(t_{e0},\omega_c)$ and $\mathbf{Y}(t_{e0},\omega_c)$ and solving for each of the successive time-samples t_{en} using the HB algorithm.

III. Comparison With Previous ETBH Technique

Despite the similarity of (4) with the expression obtained with the previously published ETHB, they are, indeed, quite different.

We should keep in mind that in the already existing ETHB method, the resulting envelope ODE is derived from the inverse Fourier transformation of a Taylor series expansion of the linear sub-network frequency characteristics around each of the harmonics of the carrier $k\omega_c$. Therefore, it does not simulate the real circuit, but a very poor reduced order model of it.

The accuracy of its results is then directly conditioned by the relation between the excitation signal spectrum bandwidth and the smoothness variation of the linear sub-network frequency characteristics inside that bandwidth. A first consequence of this is the recognized restriction of the technique to narrow bandwidth excitations as compared to the circuit characteristics [3]. But, even if this constraint is obeyed, accuracy may still be compromised whenever frequency characteristics present fast variations within the pass-band.

In fact, there are various drawbacks associated with this order model reduction approach. The first one refers to the widely recognized difficulty of using a Taylor series as an approximant. Not only it may require a large polynomial degree (which, in this case, would lead to an high order ODE) as it behaves surprisingly bad outside the approximation zone. The second problem may be even more drastic. Since there is no control of the Taylor series behavior outside the expected modulation frequency zone, it can generate approximated driving point admittances with negative real parts. And those may lead to unsuspected envelope instabilities, which will jeopardize time-domain simulations. Indeed, if the reduced order model gets unstable for a certain frequency, the time-step integration process will certainly reflect that oscillation, no matter the restrictions imposed to the envelope forcing function.

On the contrary, the present transient equation always relies on the original circuit, not suffering, therefore, from these accuracy limitations. The only problem it may face is in dealing with dispersive transmission lines, or frequency-domain element representations. Although there are various published approaches to overcome that general problem, this particular case is much simpler. Really, for most of the situations, any dispersive transmission line behaves as a simple delay to the envelope. And, for the same envelope, e.g. a microstrip discontinuity is probably well represented by a node, or by a few lumped elements, at most. So, any simple rational reduced order model, as the ones derived from asymptotic waveform evaluation [8], perfectly does the job.

IV. Illustrative Application Example

In order to show the applicability of the method, we will now present an illustrative example. The nonlinear network represented in Fig. 1 is simply a transfer nonlinearity that drives an output parallel resonant circuit. It can be viewed as a very simplified representation of an output tuned RF amplifier.

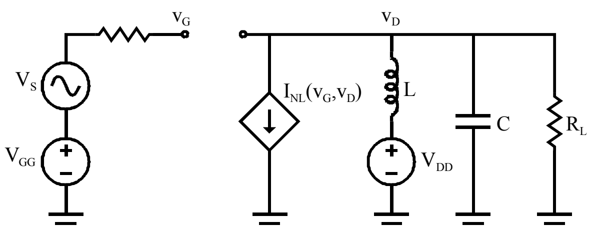

Fig. 1 – Nonlinear circuit example, for envelope transient harmonic-balance analysis.

The considered circuit excitation is the sum of a DC bias voltage (V_{GG} and V_{DD}), plus a BPSK modulated RF carrier, i.e:

$$v_s(t_e,t_c) = V_s + v_m(t_e)\left(e^{-j\omega_c t_c} + e^{j\omega_c t_c}\right) \qquad (6)$$

The carrier frequency was maintained fixed at f_c=2GHz, the output band-pass filter center frequency.

The modulating signal waveform, $v_m(t_e)$, is the pseudo-random sequence shown in Fig. 2, which leads to the eye-diagram plotted in Fig. 3.

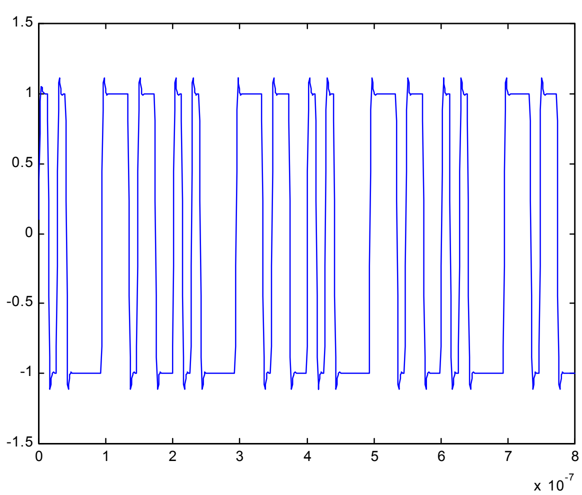

Fig. 2 – Input modulation signal waveform.

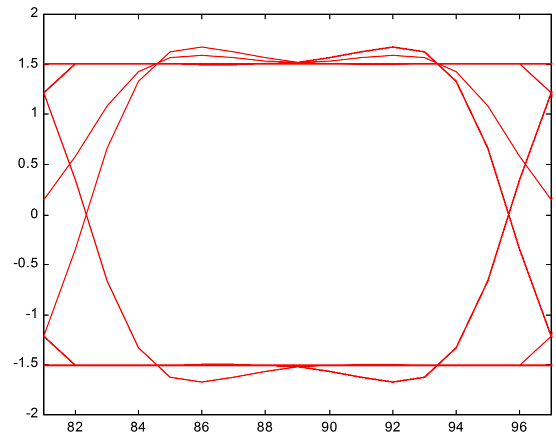

Fig. 3 – Excitation eye-diagram of the waveform shown in Fig. 2

With this signal stimulus, two different tests, corresponding to also two distinct excitation bandwidths, were simulated.

In the first case, a bit period of 133ns was used. This corresponds to a sinusoidal fundamental modulation frequency of nearly 3.75MHz, clearly below the low pass equivalent of the resonant circuit bandwidth $Bw \approx$60MHz. Therefore, this complies with the restrictions imposed by the previous ETBH. The resulting eye-diagram is plotted in Fig. 4. As expected, it is quite similar to the input eye-diagram.

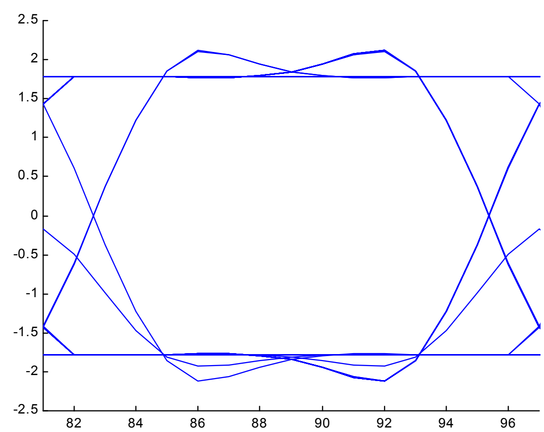

Fig. 4 – Output eye-diagram, when the envelope excitation occupies a small percentage of the circuit's bandwidth.

The second test used one tenth of the previous bit period, which locates the equivalent modulation frequency at 37.5MHz. Although this is still inside the output filter pass-band, the high frequency components of the envelope are not. The result is a significantly closed eye-diagram, as shown in Fig. 5.

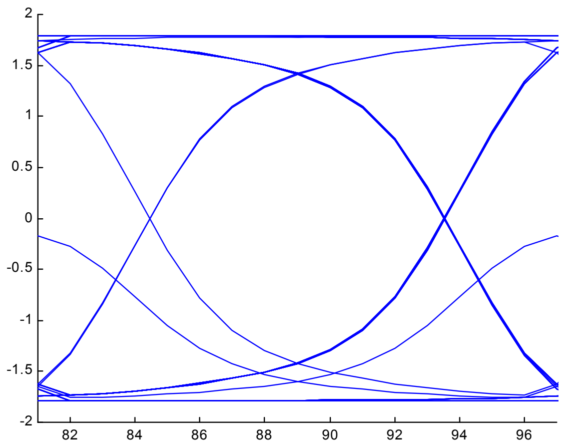

Fig. 5 – Output eye-diagram, when the envelope excitation occupies a non-negligible percentage of the circuit's bandwidth.

Fig.6 depicts the resulting simulated spectrum around de RF carrier.

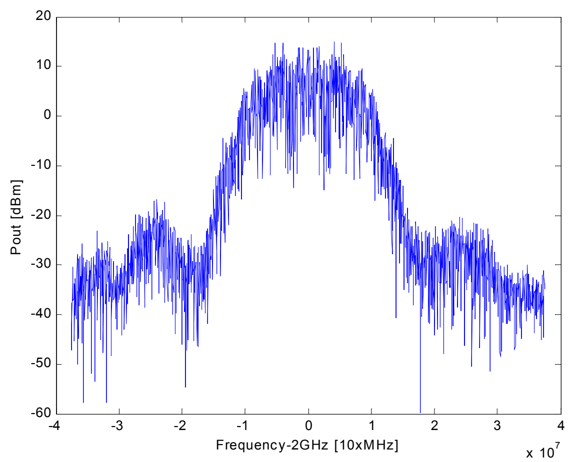

Fig. 6 – Simulated output spectrum centered at the RF carrier.

V. Conclusion

A new ETBH technique founded in the recent MPDE theory was presented. Since the only restriction it poses to the excitation is that the modulating signal cannot be time correlated with the carrier, it does not suffer from bandwidth limitations. An illustrative circuit example was then used to prove the benefit of the present method by showing that applications of more practical interest may exactly collide with the bandwidth limitations inherent to the previous ETHB technique.

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