Nonlinear Distortion Study of FM Transmission Systems Composed By a VCO-PLL Association

J. C. Pedro, N. B. Carvalho and R. C. Madureira

Instituto de Telecomunicações - Universidade de Aveiro, Portugal
email: jcpedro@gm400.det.ua.pt, nborges@trantor.it.pt, raquel@trantor.it.pt

ABSTRACT

The nonlinear distortion that arises in a FM transmission system composed by a VCO-PLL association is studied. Both, VCO-modulator and PLL-discriminator, are assumed to be nonlinear in what voltage-frequency modulation characteristics are concerned. They were represented by Volterra Functionals up to the 3rd order, from which the complete system's Functionals were derived. This model allowed analytical calculations, which showed that very linear performance can be obtained if certain conditions are met.

I. INTRODUCTION

Although there are many different analysis methods for studying the nonlinear performance of circuits and systems, this subject stills remains a very difficult task when real electronic networks are concerned. In fact, unless very simple circuits are used, analytical solutions are generally not possible. Conclusions obtained from a numerical circuit analysis are difficult to be generalised for other topologies, or even for different components' values.

A known way to circumvent these problems consists in the use of the Volterra Series Method, since it allows the circuit's nonlinear characterisation by a set of appropriate closed form formulas\cite{1,2,3}. Really, this technique trades the benefits of a nonlinear system's analytical solution, by the restriction imposed on its small excitation level, or mild nonlinearity behaviour. Fortunately, this do not represents a major drawback for many nonlinear problems found in practice, as is the case of most continuous nonlinearities subject to small signal excitation. As the FM transmission system presently under consideration, a voltage-to-frequency converter VCO, followed by a PLL frequency-to-voltage discriminator, falls into that class, the Nonlinear Volterra Functionals description of the VCO, PLL and VCO-PLL cascade, were used to evaluate this complex system's nonlinear distortion behaviour.

Studies performed with this Volterra model, showed that some interesting situations of linear operation could be obtained if certain conditions are imposed to the modulator VCO, and the one included inside the PLL discriminator.

II. VCO-PLL COMBINATION NONLINEAR VOLterra MODEL

![Fig. 1 - VCO-PLL cascaded model.](image)

The block description of the VCO-PLL cascade is represented in Fig. 1. There, the VCO is assumed to be the cascade's first block, or A, and the PLL the last, or block B. Each block is supposed to be described, in the frequency domain, by a convenient Volterra Series up to 3rd order, which Functionals, or Nonlinear Transfer Functions, NLTF's, are to be derived next using the Probing Method\cite{1,2,3}.

VCO and PLL NLTF's Derivation:

If the VCO's voltage-to-frequency conversion process is assumed to be memoriless, i.e., an input voltage variation corresponds to an immediate frequency change, the VCO transfer characteristic may be represented by a Taylor Series expansion around some input quiescent voltage, \( V_{iQ} \), like:

\[
\Omega_0(V_i,V_{iQ}) = x(V_{iQ}) + \frac{dx(V_2)}{dV_2} \bigg|_{V_{iQ}} (V_i - V_{iQ}) + \frac{1}{2!} \frac{d^2x(V_2)}{dV_2^2} \bigg|_{V_{iQ}} (V_i - V_{iQ})^2 + \frac{1}{3!} \frac{d^3x(V_2)}{dV_2^3} \bigg|_{V_{iQ}} (V_i - V_{iQ})^3 + \cdots \ (1)
\]

or in terms of incremental frequency and voltage:

\[
\omega_0(v_i) = d_1 v_1 + d_2 v_1^2 + d_3 v_1^3 + \cdots, \ (2)
\]
where $\Omega_0 = x(V_i)$ is the VCO's FM functional description. In order to maintain compatibility with the phase-voltage description adopted for the PLL discriminator, the VCO will also be treated as a voltage dependent phase modulator.

The application of a 1st order elementary exponential excitation to the time integral of (2), $v_i(t) = e^{st}$, gives a 1st order output phase response of $\theta_0^{(1)}(t) = H_M^{(1)}(s)e^{st}$. That indicates a VCO 1st order NLTF of:

$$H^{(1)}_M(s) = \frac{d_{1M}}{s}.$$  (3)

For determining the VCO's 2nd order NLTF, an elementary exponential excitation of 2nd order is considered, $v_i(t) = e^{s_1t} + e^{s_2t}$, to which corresponds an output of $\theta_0^{(2)}(t) = 2!H_2^{(2)}(s_1, s_2)e^{(s_1+s_2)t}$, and a consequent 2nd order NLTF of:

$$H^{(2)}_M(s_1, s_2) = \frac{d_{2M}}{s_1 + s_2}.$$  (4)

The derivation of the 3rd order NLTF followed exactly the same procedure, and gave:

$$H^{(3)}_M(s_1, s_2, s_3) = \frac{d_{3M}}{s_1 + s_2 + s_3}.$$  (5)

Because of its greater complexity, the NLTF's derivation of the PLL discriminator (which includes simultaneous nonlinear VCO and phase detector, PD) will be skipped, since it can be found in a previous paper of the authors.

**III. VCO-PLL Nonlinear Distortion Evaluation**

The above Volterra Model for the VCO-PLL is clearly too much complex to allow any qualitative conclusion drawn from analytical calculations. Therefore, some simplifying assumption is needed. The one chosen was that the PLL discriminator had a linear phase-detector, PD. This simplification is justified in practice by the generalised use of triangular or saw-tooth PD's.

With these considerations in mind, the system's NLTF's, above expressed by (6) to (8), become:

$$H^{(1)}(s) = \frac{c_{1d_{1M}}F(s)}{c_{1d_{1D}}}.$$  (9)

$$H^{(2)}(s_1, s_2) = \frac{d_{2M}}{d_{1M}}H^{(1)}(s_1 + s_2).$$  (10)

$$H^{(3)}(s_1, s_2, s_3) = \frac{d_{3M}}{d_{1M}}H^{(1)}(s_1 + s_2 + s_3).$$  (11)

Algebraic manipulation of these expressions predicts that the system under study is linear, i.e., has null 2nd and 3rd order NLTF's, if the VCO modulator and the VCO present at the PLL discriminator have equal voltage-frequency conversion characteristics, or:

$$d_{1M} = d_{1D}, \quad d_{2M} = d_{2D}, \quad d_{3M} = d_{3D}.$$  (12)

while the following conditions for the linear transfer functions are, simultaneously, met:

$$H^{(1)}(s_1)H^{(1)}(s_2)H^{(1)}(s_3) = 1,$$  (13)

(for zeroing 2nd order NLTF), and:

$$H^{(1)}(s_1)H^{(1)}(s_2)H^{(2)}(s_1, s_3) = 1,$$  (14)

(for zeroing 3rd order NLTF).

These latter two expressions state that the system's linearity will be a direct function of the linear frequency response flatness, and of its reduced phase delay. If one
accepts (12), then, by noting the form of (9), it is obvious that (13) and (14) are approximately verified when \(|s| \ll |c_1d_1|F(s)|. This requires a maximum FM modulation frequency, \(\omega_m\), such that \(\omega_m \ll |c_1d_1|F(j\omega_m)|. (12) through (14) define the linearity range of the system. They also describe, in mathematical terms, the intuitive idea that the system should be linear if, being the two VCO's the only nonlinearity sources, the nonlinear impairments of one compensate the nonlinear distortion of the other.

Although the use of a full model (with general PD)\(^4\) implies performing tedious calculations on (7) and (8), it can be shown that the system's linear conditions above derived for a linear PD, remain valid, even if that element is also considered nonlinear.

For each VCO-PLL combination, \(\omega_m\), the linearized system's bandwidth, strongly depends on the PLL's loop filter, LPF. Therefore, the different influences of several \(F(s)\) characteristics generally encountered in phase locked loops, were investigated.

A - Single pole loop filter
This filter has a linear transfer function of:

\[
F(s) = \frac{A}{B(s-p_1)},
\]

then:

\[
H^{(1)}(s) = \frac{c_1d_1A}{Bs(s-p_1)+c_1d_1A} = \frac{c_1d_1A/B}{s^2-p_1s+c_1d_1A/B}
\]

for this type of LPF, the PLL's natural frequency, \(\omega_n\), and damping factor \(\xi\), are such that:

\[
\omega_n = \sqrt{c_1d_1A/B} \cdot e^{2\xi\omega_n} = -p_1
\]

If the gain \(A/B\) is assumed to be an invariant \(K\), then the control of \(\xi\), in order to reduce the frequency response peaking, imposes a certain \(\omega_n\) value, and thus the system's operating bandwidth. Therefore, this type of LPF does not seem to be very attractive for present system linearization purposes.

B - Pole-zero loop filter
The usual way to circumvent the restrictions associated with the single pole LPF, is to add a zero to its transfer function. That transfer function then becomes:

\[
F(s) = \frac{A(s-z_1)}{B(s-p_1)}, \text{ and:}
\]

\[
H^{(1)}(s) = \frac{c_1d_1A(s-z_1)}{Bs(s-p_1)+c_1d_1A(s-z_1)} = \frac{c_1d_1A/B-c_1d_1z_1A/B}{s^2-(p_1+c_1d_1A/B)-c_1d_1z_1A/B}
\]

Considering again \(A/B=K\), then:

\[
\omega_n = \sqrt{c_1d_1z_1K} \cdot e^{2\xi\omega_n} = -p_1 + c_1d_1K.
\]

The presence of the two degrees of freedom, \(z_1\) and \(p_1\), allows flatness optimisation, while guaranteeing a desired loop bandwidth. However, as can be seen from Fig. 2, this \(F(s)\) still produces a high phase delay. That is the determining characteristic of the nonlinear distortion simulation's results presented in Fig. 3 and Fig. 4.

These plots compare 2nd and 3rd order two-tone nonlinear intermodulation behaviour (at \(f_1-f_2\) and \(2f_1-f_2\), respectively) of a VCO-PLL combination with equal VCO's, linear VCO and nonlinear PLL, and finally, nonlinear VCO and linear PLL. An obvious improvement in the nonlinear distortion produced by the compensated VCO-PLL combination can be obtained from DC up to about \(\omega_n\).
C - Pole-zero loop filter with pole at DC
The last studied LPF transfer function was:

\[ F(s) = \frac{A(s-z_1)}{B s}, \text{then:} \]

\[ H^{(1)}(s) = \frac{c_1 d_1 A(s-z_1)}{B s^2 + c_1 d_1 A(s-z_1)} = \frac{c_1 d_1 A s}{s^2 + c_1 d_1 A c d s} \]

for which \( \omega_n = \frac{-c_1 d_1 z_1 A}{B} e^{2\xi \omega_n} = c_1 d_1 A \).

In this case, \( \omega_n \) can be controlled with \( z_1 \). Since this is an active filter, its gain, \( A/B \), can now be used to optimise the system's linear transfer function flatness, within the required bandwidth. The system's phase delay is also reduced, as there is no term in the denominator, other than \( s^2 \), that is different from the numerator ones. The linearization condition above derived, \( B s^2 c d s A d s \), is then much more easily met. That has a direct impact on the system's nonlinear distortion characteristics, as can be observed from Fig. 6 and Fig. 7. In fact, the use of this type of LP provides an useful distortion compensation bandwidth up to about \( 2.5 \omega_n \).

![Fig. 5 - 1st order transfer function for VCO-PLL with active loop filter.](image)

![Fig. 6 - 2nd order distortion for VCO-PLL with active loop filter.](image)

IV. CONCLUSIONS
In this work it was shown how Volterra Series descriptions of a VCO modulator and a PLL discriminator, were used to study the nonlinear distortion behaviour of an FM transmission system.

The interactions between the nonlinear distortion contributions, arising from an arbitrary VCO-PLL combination, were analytically evaluated. This allowed the derivation of some general compensation conditions, which were then studied in more detail for system's implementations with loop filters usually encountered in analogue PLL's. Simulation results obtained for a passive pole-zero loop filter, and an active integrator with a zero, showed linearized transmission behaviour in a bandwidth that spanned from DC to \( \omega_n \) or \( 2.5 \omega_n \), respectively. These are believed to be promising results that encourage further research in terms of practical transparent FM or PM transmission applications.

REFERENCES