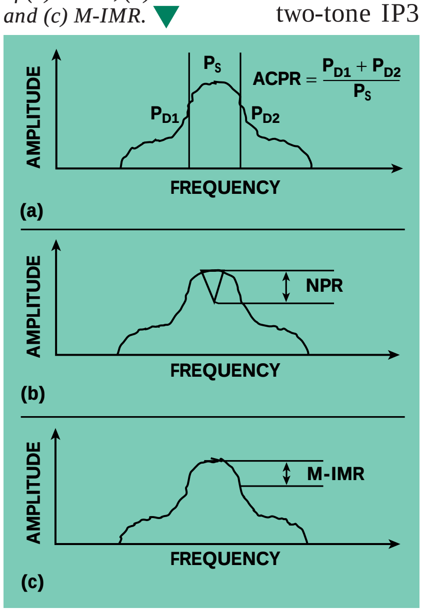
Compact Formulas to Relate ACPR and NPR to Two-tone IMR and IP3

A set of compact formulas are presented to estimate modern multitone distortion figures of merit from well-established two-tone measurements. In addition, the ability of noise power ratio (NPR) figures to predict in-band distortion of mildly nonlinear systems subject to real continuous spectra is discussed. It is shown that the usual procedure of eliminating one input tone to allow in-band distortion observation leads to an optimistic NPR value 6 dB higher than the real NPR that would be obtained if the correspondent dense spectrum was used.

Normally, when linearity behavior of an amplifier must be evaluated, the two-tone third-order intercept point (IP3) is used as a standard. However, in new telecommunications circuits (such as the ones found in modern digital mobile communications), more complex signals must be analyzed and, thus, the two-tone IP3 standard is no longer a good figure of merit. Therefore, more robust distortion marks, such as adjacent-channel power ratio (ACPR), NPR and multitone intermodulation ratio (M-IMR), must be considered. ACPR is the ratio between the total adjacent-channel integrated power and the power of the useful signal band. NPR is the ratio between the in-band distortion and useful signal spectral densities when an in-band noise spectrum slice is removed. M-IMR is the ratio between a useful tone power and the highest distortion tone power outside, but close to, the useful band. ***Figure 1*** shows graphical representations of these three parameters.

Fig. 1 Graphical definitions of (a) ACPR, (b) NPR and (c) M-IMR.



ACPR and M-IMR distortion measurements are simple extensions to the well-established two-tone intermodulation test since they relate the distortion power outside the useful band to the output signal power inside the useful band. NPR is a different type of measurement since it is intended to assess the in-band distortion. Because devices' linear output components mask in-band distortion, it is not possible to directly measure distortion products that follow on the fundamental signal unless a slice of the input signal spectrum is removed with a notch filter.

A relationship between these multitone test results and the more easily measured two-tone intermodulation distortion (IMD) would enable the full characterization of the device under multitone excitation using only the well-known two-tone intermodulation tests. Fortunately, two recently published papers[1,2] have demonstrated that these formulas can be easily deduced from some rigorous combinatory analysis.

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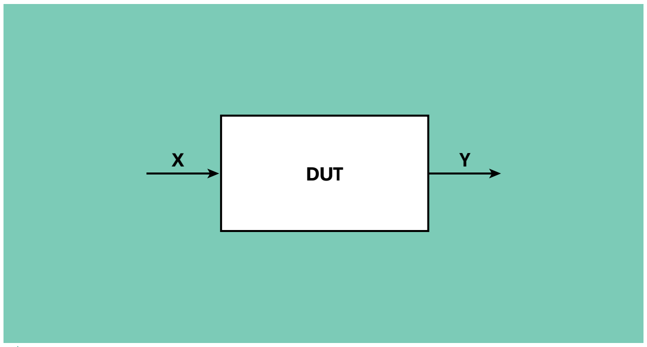


▲ *Fig. 2 The nonlinear circuit representation.*

TWO-TONE ANALYSIS

To begin with the well-established two-tone test and its figures of merit, IMR and IP3, it is assumed that the device under test (DUT) can be considered memoryless and under a small-signal excitation. This common restriction applies to almost every circuit where a large signal-to-intermodulation ratio is desired. This is the case for the major portion of so-called linear components used in modern telecommunications systems when they are driven reasonably below gain compression. Under this assumption, all nonlinear devices of practical interest can be approximately modeled by Taylor series expansions up to the third degree.[3] The device model representation is shown in ***Figure 2*** where

$$y = k_1 x + k_2 x^2 + k_3 x^3 \qquad (1)$$

Usually, RF communications circuits are narrowband. Therefore, assuming only the in-band output is of interest and that $x(t) = x_1\cos(\omega_1 t) + x_2\cos(\omega_2 t)$ where $x_1 = x_2 = x$ yields

$$\begin{aligned} y_{\text{in-band}}(t) &= k_1 x\left[\cos(\omega_1 t) + \cos(\omega_2 t)\right] \\ &+ \frac{k_3}{4}x^3\Big\{9\cos(\omega_1 t) + 9\cos(\omega_2 t) \\ &+ 3\big[\cos(2\omega_2 - \omega_1)t \\ &+ \cos(2\omega_1 - \omega_2)t\big]\Big\} \end{aligned} \qquad (2)$$

By observing Equation 2, the expected linear components can be distinguished from the new distortion components. In fact, nine new mixing products appear at ω_1 and ω_2 and three appear at $2\omega_2 - \omega_1$ and $2\omega_1 - \omega_2$. These numbers of newly generated distortion components can be generalized for any fixed mixing product using the previously published multinomial coefficient.[3]

Using Equations 1 and 2, it is possible to calculate the two-tone IP3 and IMR. The input two-tone IP3 is defined as the output power per tone required to produce equal distortion power P_{Do} at $2\omega_1 - \omega_2$ (or $2\omega_2 - \omega_1$) and linear output power P_{Lo} at ω_1 (or ω_2). To simplify calculations, consider the case of a normalized 1 Ω load resistance. Then,

$$P_{in}(\omega_1) = \frac{X^2}{2} \qquad (3)$$

IP3 implies that

$$P_{Lo}(\omega_1) = P_{Do}(2\omega_2 - \omega_1)$$

which implies that

$$\frac{k_1^2}{2}X^2 = 9\frac{k_3^2}{32}X^6$$

$$\Rightarrow X^4 = \frac{16}{9}\frac{k_1^2}{k_3^2}$$

and so

$$\begin{aligned} \text{IP3} &= P_{outo}(\omega_1) \\ &= \frac{k_1^2 X^2}{2} \\ &= \frac{1}{2}\sqrt{\frac{k_1^6}{k_3^2}\frac{16}{9}} \\ &= \frac{k_1^3}{k_3}\frac{2}{3} \end{aligned} \qquad (4)$$

The value of the two-tone IMR normalized to the total output power P_{OT} then is

$$\begin{aligned} \text{IMR}_2 &= \frac{P_L}{P_D} = \frac{\frac{k_1^2}{2}X^2}{9\frac{k_3^2}{32}X^6} \\ &= \frac{k_1^6}{k_3^2}\frac{4}{9\frac{k_1^4 X^4}{4}} = \frac{\text{IP3}^2}{P_{out}^2(\omega_1)} \\ &= \frac{\text{IP3}^2}{\frac{P_{OT}^2(\omega_1)}{4}} \\ &\Rightarrow \text{IMR2}_{dBc} \\ &= 2\left(\text{IP3}_{dBm} - P_{OT\,dBm}\right) + 6\text{ dBc} \end{aligned} \qquad (5)$$

ACPR, M-IMR AND NPR RELATIONAL FORMULAS

Following established theory,[1,2] formulas that relate general n-tone ACPR, M-IMR, NPR and co-channel power ratio (CCPR) to the derived IMR2 or IP3 are presented. These formulas enable a straightforward conversion between any pair of the distortion figures of merit.

ACPR

$$\begin{aligned} \text{ACPR}_{dBc} &= \left(\frac{P_L}{P_D}\right)_{dB} \\ &= \left(\frac{\frac{k_1^2}{2}\frac{X^2}{n}\bullet n}{\frac{k_3^2}{32}\frac{X^6}{n^3}\bullet 9\bullet 4\bullet\left(4\sum_{r=1}^{n-1}N_1 + \sum_{r=1}^{n-1}M_1\right)}\right)_{dB} \\ &= \text{IMR2}_{dBc} + 10\log\left(\frac{n^3}{16N_4 + 4M_4}\right) \end{aligned} \qquad (6)$$

where

$$N_4 = \sum_{r=1}^{n-1} N_1 = \sum_{r=1}^{n-1}\left[\left(\frac{n-r}{2}\right)^2 - \frac{\varepsilon_1}{4}\right] = \frac{2n^3 - 3n^2 - 2n}{24} + \frac{\varepsilon}{8}$$

$$M_4 = \sum_{r=1}^{n-1} M_1 = \sum_{r=1}^{n-1}\left[\left(\frac{n-r}{2}\right) + \frac{\varepsilon_1}{2}\right] = \frac{n^2 - \varepsilon}{4} \qquad (7)$$

where

$$\varepsilon = \mathrm{mod}\left(\frac{n}{2}\right)$$

$$\varepsilon_1 = \mathrm{mod}\left(\frac{n+r}{2}\right)$$

$\mathrm{mod}\left(\frac{x}{2}\right)$ = the division remainder of x by two

M-IMR

$$\text{M-IMR}_{dBc} = \left(\frac{P_L}{P_D}\right)_{dB} = \left(\frac{\frac{k_1^2}{2}\frac{X^2}{n}}{\frac{k_3^2}{32}\frac{X^6}{n^3} \bullet 9\left(4N_1 + M_1\right)}\right)_{dB} = \mathrm{IMR2}_{dBc} - 6 + 10\log\left(\frac{n^2}{4N_1 + M_1}\right) \qquad (8)$$

where

$$N_1 = \left(\frac{n-r}{2}\right)^2 - \frac{\varepsilon_1}{4}$$

$$M_1 = \left(\frac{n-r}{2}\right) + \frac{\varepsilon_1}{2}$$

In this case, N_1 and M_1 are calculated for r = 1, that is, where the highest distortion tone power outside the useful band is located.

Usual NPR

$$\mathrm{NPRu}_{dBc} = \left(\frac{P_L}{P_D}\right)_{dB} = \left(\frac{\frac{k_1^2}{2}\frac{X^2}{n}}{\frac{k_3^2}{32}\frac{X^6}{n^3} \bullet 9\left(4N_2 + M_2\right)}\right)_{dB} = \mathrm{IMR2}_{dBc} - 6 + 10\log\left(\frac{n^2}{4N_2 + M_2}\right) \qquad (9)$$

where

$$N_2 = \left(\frac{n-b-2}{2}\right)^2 - \frac{\varepsilon_2}{4} + \left(\frac{b-1}{2}\right)^2 - \frac{\varepsilon_3}{4} + b\left(n-b-2\right) \qquad (10)$$

$$M_2 = \left(\frac{n-b-2}{2}\right) + \frac{\varepsilon_2}{2} + \left(\frac{b-1}{2}\right) + \frac{\varepsilon_3}{2} \qquad (11)$$

where

$$\varepsilon_2 = \mathrm{mod}\left(\frac{n+b}{2}\right)$$

$$\varepsilon_3 = \mathrm{mod}\left(\frac{b+1}{2}\right)$$

This case is called the usual NPR since it is the measurement usually performed in the laboratory: The circuit is excited with a multitone signal with one middle tone shut down in order to reveal the distortion components present.

CCPR

The CCPR test is what ideally could be done if the input signal consisted of a multitone spectrum without any tone shut down (a situation much closer to the normal nonlinear system's operation). Contrary to the previously discussed NPR, the distortion on that particular position is measured in the presence of the corresponding input spectral line.

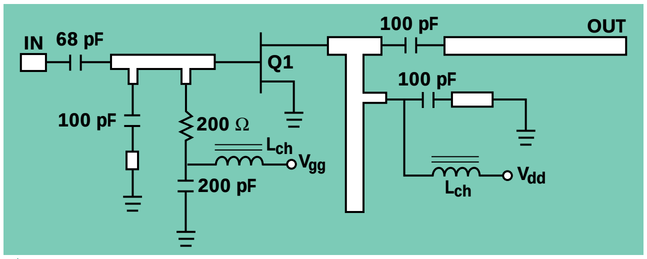


Fig. 3 The class A power amplifier circuit schematic used in the application examples.

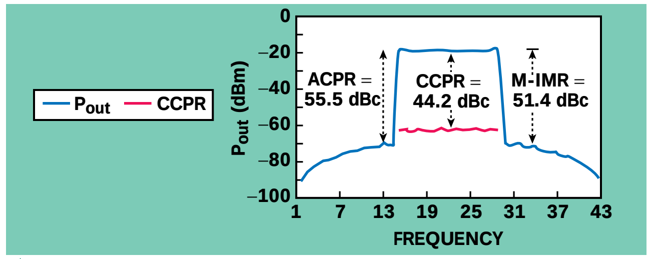


Fig. 5 Output distorted spectrum and CCPR measurement.

Fig. 4 Harmonic balance-simulated two-tone P_{out} and IMD, and extrapolated IP3.

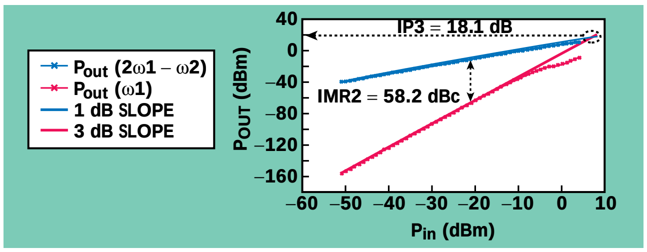
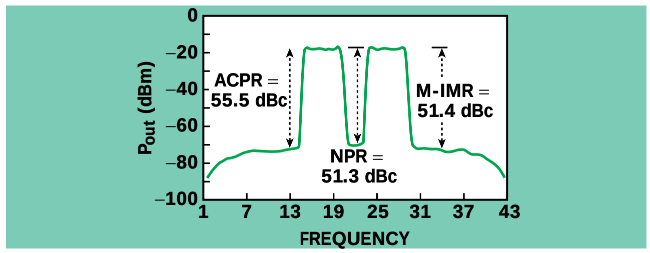


Fig. 6 Output distorted spectrum and traditional NPR measurement.



$$\begin{aligned} CCPRr_{dBc} &= \left(\frac{P_L}{P_D}\right)_{dB} \\ &= \left(\frac{\dfrac{k_1^2}{2}\dfrac{X^2}{n}}{\dfrac{k_3^2}{32}\dfrac{X^6}{n^3} \bullet 9\left(4N_3 + M_3 + \dfrac{S^2}{9}\right)} \right)_{dB} \\ &= IMR2_{dBc} - 6 + 10\log\left(\frac{n^2}{4N_3 + M_3 + \dfrac{S^2}{9}} \right) \end{aligned} \quad (12)$$

where

$$N_3 = \left(\frac{n-b-2}{2}\right)^2 - \frac{\varepsilon_2}{4} + \left(\frac{b-1}{2}\right)^2 - \frac{\varepsilon_3}{4} + b(n-b-2) + b \quad (13)$$

$$M_3 = \left(\frac{n-b-2}{2}\right) + \frac{\varepsilon_2}{2} + \left(\frac{b-1}{2}\right) + \frac{\varepsilon_3}{2} \quad (14)$$

$$S = (6n-3) \quad (15)$$

A comparison between Equations 12 through 15 and 9 through 11 reveals that significantly more mixing terms now appear. Therefore, the distortion power in this case is indeed much worse than that obtained from the usual NPR. This result is a clear indication that the traditional way of measuring NPR only provides a first and optimistic estimate of the real in-band distortion generated by the nonlinear component subject to a dense input spectrum.[1,2] In fact, any simple combinatory analysis will show that the intuitive thought that shutting down only one tone from a large number of input spectral lines will have a negligible impact on the measurement is indeed false. Actually, when the single tone ω_i is shut down, there will be at least 6(n – 1) mixing products of the form $\omega_i + \omega_j - \omega_j$ and three of the form $\omega_i + \omega_i - \omega_i$ eliminated.[1,2]

TABLE I

SIMULATED AND CALCULATED RESULTS

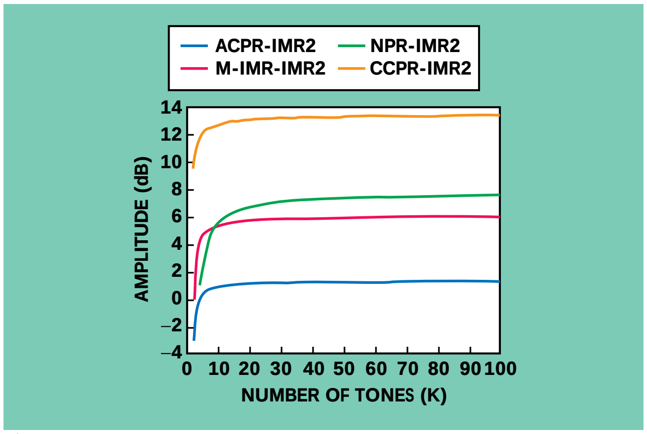
	ACPR (dBc)	M-IMR (dBc)	NPR (dBc)	CCPR (dBc)
Simulated	55.5	51.4	51.3	44.2
Calculated	55.7	51.1	51.2	44.0

APPLICATION EXAMPLES

Consider the microwave power amplifier shown in ***Figure 3***, biased for class A operation. This power amplifier was excited by a two-tone test. The simulated output power at ω_1 and in-band IMD at $2\omega_2 - \omega_1$ with an in-house harmonic balance engine[4] are shown in ***Figure 4***.

IP3 can be obtained as the interception of the extrapolated IMD 3 dB slope line and the linear output power line of 1 dB slope. In this case, IP3 = 18.1 dBm. The power amplifier was then excited with a 15-tone input spectrum at a total input power level of –8.2 dBm, equal to the equivalent two-tone input power for an IMR2 = 58.2 dBc. The amplifier's simulated[4] output response is shown in ***Figure 5***. ***Figure 6*** shows results that are similar but now correspondent to a traditional NPR test where three input tones were shut down.

Using the ACPR, M-MIR, NPR and CCPR values derived from the graphs and the calculated results obtained with the expressions presented in previous sections, ***Table 1*** lists the data for comparison. As can be seen, fast and accurate results for multitone figures of merit can be easily obtained using the presented formulas. For example, for the case just presented, the required simulation time was five hours against immediate results provided by the formulas.



▲ *Fig. 7 Normalized relationship between multitone ACPR, M-IMR, NPR, CCPR and IMR2 vs. the number of input tones.*

Example 1

Now consider the same power amplifier (IP3 = 18.1 dBm and gain G = 11 dB), where a design specification of 40 dBc ACPR under a 100-tone excitation is required. The design goal is the input power level per tone in order to meet the ACPR specifications. Using Equations 5 and 7, N_4 and M_4 for 100 tones become $N_4 = 82075$ and $M_4 = 2500$. Thus,

$$\begin{aligned}
\text{ACPR} &= 40 \text{ dBc} \\
&= \text{IMR2} + 10\log\left(\frac{100^3}{16 \bullet 82075 + 4 \bullet 2500}\right) \\
&= \text{IMR2} - 1.2 \\
&= 2\left(\text{IP3} - P_{OT}\right) + 6 - 1.3 \\
\Rightarrow P_{OT} &= \frac{36.2 - 1.2 + 6 - 40}{2} = 0.5 \text{ dBm} \\
\Rightarrow P_{in}(\text{tone}) &= \frac{P_{inT}}{n} = \frac{P_{OT}}{nG} \\
&= P_{OTdBm} - 10\log(n) - G_{dB} \\
&= 0.5 - 20 - 11 = -30.5 \text{ dBm}
\end{aligned}$$

If an alternative specification of M-IMR = 40 dBc were the goal, then using Equations 5 and 8 yields $N_1 = 2450$ and $M_1 = 50$.

$$\begin{aligned}
\text{M-IMR} &= 40 \text{ dBc} \\
&= \text{IMR2} - 6 + 10\log\left(\frac{100^2}{4N_1 + M_1}\right) \\
&= 2\left(\text{IP3} - P_{OT}\right) + 6 - 6 + 0.06 \\
\Rightarrow P_{OT} &= -1.9 \text{ dBm} \\
\Rightarrow P_{in}(\text{tone}) &= \frac{P_{OT}}{nG} = -29.1 \text{ dBm}
\end{aligned}$$

Now consider the co-channel interference problem. If the amplifier were specified for the same 40 dBc using a traditional NPR test on the 51st tone, then according to Equations 9 through 11, $N_2 = 3577$ and $M_2 = 49$.

$$\begin{aligned}
\text{NPRu} &= 40 \text{ dBc} \\
&= \text{IMR2} - 6 + 10\log\left(\frac{100^2}{4N_2 + M_2}\right) \\
&= 2\left(\text{IP3} - P_{OT}\right) - 1.6 \\
\Rightarrow P_{OT} &= -2.7 \text{ dBm} \\
\Rightarrow P_{in}(\text{tone}) &= -33.68 \text{ dBm}
\end{aligned}$$

If, on the other hand, a similar CCPR of 40 dBc were considered but the 51st tone is no longer shut down, the input power using Equations 12 through 15 would now have to be $N_3 = 3626$, $M_3 = 49$ and $S = 597$.

$$\begin{aligned}
\text{CCPRa} &= 40 \text{ dBc} \\
&= \text{IMR2} - 6 + 10\log\left(\frac{100^2}{4N_3 + M_3 + \frac{S^2}{9}}\right) \\
&= 2\left(\text{IP3} - P_{OT}\right) - 7.3 \\
\Rightarrow P_{OT} &= -5.6 \text{ dBm} \\
\Rightarrow P_{in}(\text{tone}) &= -36.5 \text{ dBm}
\end{aligned}$$

As can be seen, there is a difference of more than 3 dB in input power when the CCPR or usual NPR is considered. Thus, if the circuit is excited with –33 dBm per tone (as would have been done if the usual NPR approach was considered), CCPR = 32.9 dBc would only be produced under normal operation, which is more than 7 dB below the 40 dBc goal.

Example 2

Consider now a plot of ACPR-IMR2, M-IMR-IMR2, NPR-IMR2 and CCPR-IMR2 vs. the number of input tones n. For n > 10 a good approximation of a continuous spectrum is obtained. Thus, the multitone input spectrum with constant total power becomes a good discretized version of the real signal power spectral density. ***Figure 7*** shows that the usual NPR is approximately 6 dB below the CCPR. Therefore, if the usual NPR measurement procedure is to be used to estimate distortion arising from an input-dense spectrum, a 6 dB offset in distortion must be added to the observed measurement results.

The values presented here for M-IMR have already been demonstrated by measurements in previously published work.[3] Calculating the limits of Equations 6 through 15 when n tends toward infinity gives ACPR-IMR2 = 4.3 dB, M-IMR-IMR2 = 6.0 dB, NPR-IMR2 = 7.8 dB and CCPR-IMR2 = 13.4 dB. Comparing the results of CCPR-IMR2 and NPR-IMR2, a 5.6 dB increase in the middle of the input band must be considered when a usual NPR is measured.

CONCLUSION

A rigorous analysis of the distortion behavior of a third-order memoryless nonlinear system subject to a multitone spectrum has been presented. The derived expressions allow closed-form relationships between well-established two-tone results and modern multitone figures of merit. This analysis also showed that the usual NPR measurement procedure provides an optimistic estimate of the distortion that arises in a real nonlinear system driven by a multitone or even continuous spectrum. However, it also can be shown that, under the considered conditions, the addition of a constant correction factor of 6 dB is sufficient to produce accurate results. ■

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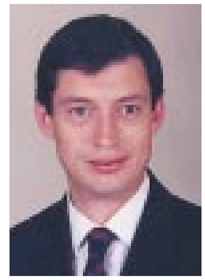

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