A critical study of the open-ended coaxial line sensor technique for RF and microwave complex permittivity measurements

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thin cover glass substrates. This technique, which is almost as sensitive as an interferometric cantilever method, requires simpler and cheaper components and could easily be automated. As the use of thin-film electrodes is expanding for several electrochemical devices, it is likely that the set-up presented in this paper could find application in many other experiments.

Acknowledgments
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A critical study of the open-ended coaxial line sensor technique for RF and microwave complex permittivity measurements

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Abstract. An HP 8510 automatic network analyser and a six-port reflectometer have been used with a purpose-built calculable open-ended coaxial line sensor to measure the reflection coefficients of various materials, including dielectric reference liquids, in the frequency range 50 MHz–2.0 GHz. Factors crucial for calculable measurements have been identified including associated measurement uncertainties. The reference material measurements have been used in critical studies of (i) a commonly employed lumped equivalent circuit model of the fringing fields of the sensor, and (ii) a numerical point-matching theory of the propagating and evanescent modes at the termination of the sensor. The equivalent circuit model is shown, both theoretically and experimentally, to be generally inadequate for the full range of complex permittivities and frequencies which the sensor could otherwise cover. The point-matching theory has been used to predict the reference material reflection coefficients and the agreement with the expected behaviour measured for deionised water is to within 0.007 and 0.7° for the magnitude and phase of the reflection coefficient, respectively. An inverse solution based on the point-matching theory has been developed to derive complex permittivity from reflection coefficients and it has been used to measure various polar liquids. We conclude that the inverse point-matching theory should enable a more widespread and accurate exploitation of the sensor technique for various applications including those in biomedicine and industrial quality control.

1. Introduction
Measurements of the complex permittivity of materials as functions of frequency and temperature provide information of importance in various fields of physical chemistry, engineering, industry and medicine. Complex permittivity data of biological tissues are required for medical research into more effective electromagnetic (EM) hyperthermia treatments (Armitage et al 1983, Pethig 1984). Recent work, moreover, suggests that the complex permittivity may be of value for tissue characterisation and diagnostic imaging using non-ionising EM radiation (Grant 1984, Grant and Spyrou 1985).

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Other applications include the quality control of semi-solid products including food materials (Kent 1985). Of the techniques that have been developed to produce accurate radiofrequency (RF) and microwave (MW) dielectric measurements (Kaatz and Giese 1980, Birch and Clarke 1982, Afsar et al 1986) many are inapplicable to polar and biological materials which have relatively large loss-tangents. With suitable precautions, bridge and time-domain techniques can be applied to these materials (Schwan 1963, Hart 1982), yet they nevertheless require the specimen to be cut or shaped to fit a particular sample-holder geometry. These techniques are also typically very sensitive to the parameters of the experimental apparatus; e.g. the elimination of effects due to stray fields in the RF bridge measurements can often be a severe problem. Such techniques do not therefore provide the means for both accurate and convenient non-destructive RF and MW permittivity measurements.

There is increasing interest in the application of the open-ended coaxial line for convenient and non-intrusive complex permittivity measurements of lossy materials at RF and MW frequencies; determination of the reflection coefficient at the sensor discontinuity plane allows one to derive the complex permittivity of a liquid or semi-solid sample terminating the line (Stuchly and Stuchly 1980). The measurement configuration of the coaxial line sensor with a ground plane is shown in figure 1. The automatic network analyser (ANA) has so far been the instrument most frequently used to determine reflection coefficients at the sensor discontinuity plane (Athey et al 1982). The complex permittivity has commonly been extracted from the reflection coefficient by modelling the fringing fields at the discontinuity as an equivalent lumped admittance (Stuchly et al 1974, Stuchly et al 1982b). The equivalent circuit parameters of the sensor have been inferred from measurements on 'known' dielectrics and then used to calculate the complex permittivity of an unknown sample. A frequent assumption of this approach has been that the fringing fields in the sample produce a capacitance that is linearly proportional to sample relative permittivity and independent of both frequency and sample loss. A more complete analysis, taking account both of the production of higher order modes at the discontinuity and of the propagation of radiation (Mosig et al 1981), suggests that this simple approach can produce significant errors that are functions of complex permittivity and that are thus, for unknown materials, difficult to quantify.

In this study, which commenced in 1981, we examine in detail the theoretical basis of the coaxial line sensor technique and we develop improved methods of obtaining the complex permittivity with calculable uncertainties. Towards these goals (i) a purpose-built sensor has been developed for complex permittivity measurements of lossy materials under optimum capacitance conditions, (ii) a 6-port reflectometer and (iii) a state-of-the-art automatic network analyser (HP 8510) have been used for the reflection coefficient measurements, (iv) the equivalent circuit model has been examined theoretically and experimentally as a means of deriving complex permittivity and (v) the point-matching theory of Mosig et al (1981) has been examined theoretically and experimentally, largely corroborated, and extended to an inverse solution. Finally, we have used these instruments and the inverse solution to realise RF and MW complex permittivity measurements of various lossy materials.

2. Principles
There have been two basic approaches to the determination of complex permittivity from measurements of the coaxial line open-circuit reflection coefficient: (i) methods that have used a lumped equivalent circuit description of the sensor's fringing fields and (ii) methods that attempt a rigorous solution of the electromagnetic field equations appropriate for a coaxial line open to a dielectric sample. The relative simplicity of the former treatment accounts for its greater popularity (Athey et al 1982) and the latter approach has been largely restricted to a theoretical development of the 'forward' problem, \( e^*\rightarrow \Gamma^* \), where \( e^* = e' - je'' \) is the specimen complex permittivity and \( \Gamma^* \) is the discontinuity reflection coefficient; i.e. to a derivation of reflection coefficients from known permittivities (Mosig et al 1981). Later we describe an inverse solution, \( \Gamma^* \rightarrow e^* \), based on a point-matching (rms) theory. We first introduce in more detail each of these methods.

2.1. Equivalent circuit method
A method of calculating the complex permittivity that has the advantage of yielding simple closed-form equations is based on a model of the discontinuity at the termination of the coaxial line as an equivalent lumped circuit (Stuchly et al 1974, Rzepecka and Stuchly 1975). This discontinuity, in the absence of a lossy dielectric, is frequently assumed to be purely capacitive and to consist of fields fringing both within the sensor's PTFE (Teflon) dielectric and out in the sample material terminating the line (e.g. Athey et al 1981). The sensor discontinuity is then modelled as a lumped admittance with capacitances originating from the PTFE fringing field \( C_s \) and sample fringing field \( C_t \), as shown in figures 2(a) and

![Figure 1. Open circuit coaxial line sample measurement configuration.](image)

![Figure 2. (a) Coaxial line termination fringing fields. (b) Sensor lumped equivalent circuit.](image)
(b). The load admittance, \( Y_t \), is given in terms of the voltage reflection coefficient, \( \Gamma \), as
\[
Y_t = Y_o (1 - \Gamma^*)/(1 + \Gamma^*)
\]
where \( Y_o \) is the characteristic admittance of the line. In terms of the equivalent circuit model, the admittance is
\[
Y_t = j\omega C_t + j\omega (\varepsilon' - \varepsilon^*) C_t
\]
where \( \omega \) is the angular frequency. Thus,
\[
\varepsilon' = -\frac{2\Gamma}{\omega C_z(1 + 2\Gamma \cos\varphi + |\Gamma|^2)}
\]
and
\[
\varepsilon'' = \frac{1 - |\Gamma|^2}{\omega C_z(1 + 2\Gamma \cos\varphi + |\Gamma|^2)}
\]
where \( |\Gamma| \) and \( \varphi \) are the magnitude and phase of the reflection coefficient, respectively, and \( Z_o \) is the characteristic impedance. We can also define the load admittance as
\[
Y_t = j\omega C_f^\varepsilon
\]
where \( C_f^\varepsilon \) is a total complex fringing capacitance. Thus
\[
C_f^\varepsilon = (C_e^\varepsilon - C_t) - jC_e^\varepsilon.
\]
Expressions giving the approximate uncertainties in \( \varepsilon' \) and \( \varepsilon'' \) as a function of the uncertainties in \( C_t, Z_o, |\Gamma| \) and \( \varphi \) have been derived (Athey et al. 1982) as
\[
\frac{\Delta \varepsilon'}{\varepsilon'} = \left[ \frac{(\Delta C_t)}{C_t} + \frac{(\Delta Z_o)}{Z_o} \right]^2 + \left[ \frac{1 - |\Gamma|^2}{1 + 2\Gamma \cos\varphi + |\Gamma|^2} \right]^2 \left[ \frac{\Delta |\Gamma|}{|\Gamma|} \right]^2
\]
and
\[
\frac{\Delta \varepsilon''}{\varepsilon''} = \left[ \frac{(\Delta C_t)}{C_t} + \frac{(\Delta Z_o)}{Z_o} \right]^2 + \left[ \frac{4|\Gamma| + 2 \cos \varphi + 2|\Gamma|^2 \cos \varphi}{(1 - |\Gamma|^2)(1 + 2\Gamma \cos\varphi + |\Gamma|^2)} \right]^2 \left[ \frac{\Delta |\Gamma|}{|\Gamma|} \right]^2
\]
where equation (7) is here corrected (M A Stuchly, private communication) from that previously published. Differentiation of equations similar to (7) and (8) leads to an expression for the optimum sensor capacitance (Rzepecka and Stuchly 1975) as
\[
C_s = \frac{1}{\omega Z_o(\varepsilon'^2 + \varepsilon''^2)^{1/2}}
\]
provided that \( \Delta \varphi = \Delta |\Gamma|/|\Gamma| \).

Equation (3) can be rearranged in the following way:
\[
\frac{-2\Gamma \sin \varphi}{\omega Z_o(1 + 2\Gamma \cos\varphi + |\Gamma|^2)} = C_t \varepsilon' + C_t \varepsilon''
\]
In this form, \( C_t \) and \( C_t \) can be found from the gradient and ordinate intersection, respectively, of plotted data of the reflection coefficients of 'known' dielectric reference materials. The equivalent circuit model is valid providing (i) \( C_t \) and \( C_t \) are independent of sample complex permittivity, (ii) \( C_t \) and \( C_t \) are independent of frequency and (iii) the probe does not launch propagating radiation (i.e. it does not behave as an antenna).

2.2. Point-matching method
In the approach of Mosig et al. (1981) the reflection coefficient of the fundamental TEM mode in the coaxial line is obtained numerically by matching the electromagnetic fields at the interface \((z = 0) \) in figure 1) between the coaxial line and the external dielectric.

The fields inside the line are expressed in terms of a superposition of the forward travelling TEM wave, its reflection (from the interface) and a series of reflected, but evanescent, TM modes (those TM modes which preserve cylindrical symmetry) in the form:
\[
E_p = U_0 \left( f_0(\rho) \exp(-\gamma_0 \rho) + \sum_{n=0}^{\infty} R_n f_n(\rho) \exp(\gamma_n \rho) \right)
\]
and
\[
H_0 = j\omega \epsilon_0 \mu_0 U_0 \left( f_0(\rho) \gamma_0^{-1} \exp(-\gamma_0 \rho) - \sum_{n=0}^{\infty} R_n f_n(\rho) \gamma_n^{-1} \exp(\gamma_n \rho) \right)
\]
where \( U_0 \) is an arbitrary amplitude factor, \( R_n \) is the generalised reflection factor of mode \( n \) (so that \( R_0 = \Gamma \)), \( \epsilon_0 \) is the permittivity of free-space \((\epsilon_0 = 8.854 \times 10^{-12} \text{ F m}^{-1}) \) and the propagation factor of mode \( n \) is given by
\[
\gamma_n = \left| p_n^2 - \epsilon_n(\omega/c_0) \right|^{1/2}
\]
where \( c_0 \) is the speed of light in free space. Here, the radial functions \( f_n(\rho) \) are given by
\[
f_n(\rho) = \frac{N_n}{\rho}
\]
with the normalising factor
\[
N_n = \left| \ln(b/a) \right|^{-1/2}
\]
and, for \( n > 0 \),
\[
f_n(\rho) = N_n [J_n(p_n a)Y_n(p_n a) - Y_n(p_n a) J_n(p_n a)]
\]
where \( J_n(x) \) and \( Y_n(x) \) are Bessel functions, of the first and second kinds respectively, of order \( m \). In the latter case, the normalising factor \( N_n \) is given by
\[
N_n = \pi p_n [2J_n(p_n a)/J_n(p_n b) - 1]^{1/2}
\]
while the eigenvalues \( p_n (n > 0) \) are obtained by solving
\[
Y_n(p_n a) J_n(p_n b) = J_n(p_n a) Y_n(p_n b).
\]
For the dominant TEM mode, which is the only one which is not evanescent, \( p_n = 0 \) so that
\[
\gamma_0 = |\epsilon_0|^{1/2}(\omega/c_0)
\]
from equation (13).

In the exterior dielectric \((z > 0) \) in figure 1) the magnetic field is given, in terms of the electric field \( E_p \) at the interface (aperture), by
\[
H_0(\rho, z) = j\epsilon_0(\omega/c_0)^2(2\pi \mu_0 \omega)^{-1}
\]
\[
\times \int_0^\infty \int_0^{2\pi} E_p(\rho') \exp[-j(\omega/c_0)(\rho'^2 + z^2)^{1/2}] r^{-1} \rho' \cos \psi \, d\rho \, d\psi
\]
with
\[
\psi = \varphi - \varphi'
\]
and
\[
r = (\rho^2 + \rho'^2 - 2\rho \rho' \cos \psi + z^2)^{1/2}
\]
where \((\rho', \varphi')\) are the transverse (polar) coordinates of the source point within the aperture and \(r\) is the distance from the source to the field point \((\rho, \varphi, z)\).

Assuming a continuity of \(E_\rho\) at the interface, the continuity of the magnetic field \(H_\rho\) reduces to the form

\[
\sum_{n=0}^{N} R_n T_n = 1
\]  

(23)

where

\[
T_n = \left[ f_n(\rho) \gamma_n + (e^* / \epsilon_0) I_n \right] / \left[ f_n(\rho) \gamma_0 - (e^* / \epsilon_0) I_0 \right] \]  

(24)

in which

\[
I_n = \frac{1}{(2\pi)} \int_{\rho}^{\rho'} \int_{0}^{2\pi} \int_{0}^{\pi} \exp[-j(\omega/\epsilon_0)(e^*)^{1/2}r] r^{-1} \cos\varphi \, d\gamma \, d\rho'
\]  

(25)

with \(z=0\).

Since the higher-order modes \(TM_n\) are evanescent, the terms of the series in equations (11) and (12) decay rapidly as \(n\) increases. Consequently, these series may be truncated after a small number \(N\) of terms and, correspondingly, equation (23) becomes

\[
\sum_{n=0}^{N} R_n T_n = 1
\]  

(26)

In the point-matching method, equation (26) is applied at \(N\) distinct points, corresponding in the present case to the radii

\[
\rho_i = a + [(b - a)/2] [(i - 1)/ (N - 1) + (2i - 1)/2N]
\]  

\(i = 1, 2, \ldots, N\),

(27)

and the resulting set of \(N\) simultaneous linear algebraic equations is solved for the reflection coefficients \(R_n\). \(n = 1, 2, \ldots, N\). The integrals \(I_n\) are evaluated numerically, after subtraction from the integrand of the singularity at \(r = 0\). The reflection coefficient \(R_O\) of the dominant TEM mode is found to be relatively insensitive to the number of higher-order modes considered so that \(N = 5\) is taken to be sufficient in the present work.

3. Instrumentation

3.1. The six-port reflectometer

We were prompted by developments in six-port reflectometer theory and instrumentation (Engen 1977) to investigate whether the apparent advantages of relative simplicity, stability and low cost of the six-port might allow a wider exploitation of the coaxial line sensor technique than had been possible from \(\text{ANAs}\). Thus, although the measurements presented below were obtained with an HP 8510 \(\text{ANA}\) (described in §3.2) it was an important early aim of this work to study the capabilities of a six-port/dielectric-sensor combination (Grant 1984); this would seem to offer greater potential in the long term for inexpensive and perhaps portable measurement equipment (Zurcher et al. 1983). The six-port reflectometer used in this initial work was designed and constructed by Granville-George and Woods (1981) following theory devised by Woods (1979, 1983). The estimated uncertainty for measurements on this instrument was \(\pm 0.004\) for \(\Gamma\) and \(\pm 0.4^\circ\) for phase \((0.5 < |\Gamma| < 1.0)\) in the frequency range 100 MHz to 1 GHz.

3.2. The HP 8510A automatic network analyser

This instrument was employed for measurements every 10 MHz from 50 MHz to 2.0 GHz. It was found that the calibration strategy significantly influenced measurement accuracy. Calibrations were best achieved in the \(\text{ANA}\) ‘stepped’ mode using open and short circuits and matched loads below 1 GHz and offset short circuits above 1 GHz. As might be expected, measurements on this instrument proved to be far more conveniently and rapidly performed than on the six-port reflectometer. As a diagnostic tool it was also able to reveal systematic effects which were not immediately apparent when using the six-port at a limited set of frequencies; e.g. resonances in the body of the dielectric (§4.2). However, such an expensive instrument cannot reasonably be expected to be generally available on a continuous basis. The development of a dedicated six-port technique should therefore remain a major goal.

The manufacturer’s stated measurement uncertainty for the HP 8510 \(\text{ANA}\) when calibrated in the 7 mm line (as opposed to the 14 mm line used here) is better than \(\pm 1.5\%\) for a reflection coefficient modulus of 1.0 and better than \(\pm 0.8^\circ\) in phase. The performance must depend upon the standards used for calibration and the care with which they are used. In the event, 95% confidence level uncertainties of 0.005 for \(|\Gamma|\) when \(0.5 < |\Gamma| < 1.0\), and \(0.5^\circ\) for phase were obtained when employing 14 mm line standards. The chief contributions to the uncertainty arise from a lack of knowledge of both the fixed load used to calibrate the system below 1 GHz, and of the low-frequency fringing capacitance of the 14 mm shielded open-circuit capacitance (0.169 ± 0.003 pF); this latter factor is due entirely to the uncertainty about the position of its collet.

3.3 The dielectric sensor

Considerations of the optimum sensor capacitance for RF and MW polar liquid and biological material complex permittivity measurements (equation (9)) indicated that a value of order 0.1 pF for the fringing capacitance in air provides a reasonable compromise for the frequency range 50 MHz-2.0 GHz. The sensor capacitance values of commercially available \(\text{PTEF}\) coaxial lines have been listed (Athey et al. 1982) and are generally smaller than this which makes them particularly unsuitable at the lower frequencies. For this work, a 50 \(\Omega\) 14 mm diameter brass airline with GPC-14 (GR900) compatible \(\text{PTEF}\) dielectric connectors was used to form the basis of a purpose-built sensor with a near-optimum value of fringing field capacitance. The measurement configuration is shown in figure 3. To form the desired geometry, one of the GPC-14-compatible terminating connectors was machined to a flat surface through the \(\text{PTEF}\) bead and fitted with a 5 cm diameter flange to provide a ground-plane; this formed the sensor/sample interface, as shown in figure 4. The flange was
used to ensure compatibility with the geometry assumed by the point-matching theory. Semirigid, rather than flexible, coaxial cable was used to connect the probe to the measurement port of the ANA or six-port as it was found that repeatability was enhanced in this way.

When compared to many sensors that have been previously used (Stuchly and Stuchly 1980) the present probe possesses the further advantage that the GPC-14 sensor end of the airline may be unscrewed and replaced by an intact GPC-14-compatible connector whose termination provides a convenient calibration reference plane. Calibration with respect to this plane necessitates only a small correction (0.4%) for the short length of transmission line that corresponds to the difference between the sample measurement and calibration planes. This is in contrast to the previous work with smaller commercial coaxial lines where the calibrations have necessarily been done at the more remote ANA test-port plane.

4. Probe standardisation

4.1. Measurement plane offset correction

To fully standardise the probe assembly for complex permittivity measurements it was necessary first to analyse the effect of the difference between the sensor measurement and calibration planes, shown in figure 4. The reflection coefficient measured by the ANA or six-port reflectometer with respect to plane C-C', \( \Gamma_c \), must be transformed to that of the load at plane B-B', \( \Gamma_L \). The transmission line separating these planes may be treated as a two-port network through the standard relation

\[
\Gamma_L = S_{11} + S_{12} \Gamma_c + S_{21} \Gamma_c + S_{22} \Gamma_c^2
\]

where \( S_{ij} \) are the scattering parameters of the transmission line and \( \Gamma_c \) is the reflection coefficient at the calibration plane. For a perfectly matched (lossless) transmission line, \( S_{11} = S_{22} = 0 \), and for a linear, passive, two-port network, \( S_{11} = S_{12} = T \). Thus

\[
\Gamma_L = T^2 \Gamma_c
\]

in which \( T \), representing the phase change due to the offset between planes B-B' and C-C', is given by

\[
T = \exp(j\beta d)
\]

where \( \beta \) is the phase constant and \( d \) is the distance between the measurement and calibration planes. Therefore

\[
\Gamma_L = \exp(-j\beta d) \Gamma_c
\]

or

\[
\Gamma_L = |\Gamma_c| \exp[j\varphi - 4\pi f(L_1 - L_2)/c]
\]

where \( f \) is the frequency and \( c \) is the velocity of light. With \( L_1 \) and \( L_2 \) measured, respectively, as 32.13 ± 0.03 mm and 20.24 ± 0.03 mm, \( c = 2.9979 \times 10^8 \text{ mm s}^{-1} \), and \( f \) expressed in MHz, equation (32) reduces to

\[
\Gamma_L = |\Gamma_c| \exp[j\varphi - (0.0285 ± 0.0002)f]
\]

where the phase angles are in degrees. The validity of equation (33) was tested by measurements of the sensor short-circuited at plane B-B' (by a firm placement of the sensor against aluminium foil on a rubber pad). The maximum scatter in \( |\Gamma| \) was ±0.004 with the observed errors in \( \varphi \) increasing very slightly with frequency to a maximum of ±0.5°.

4.2. Dielectric reference materials

A range of dielectric reference materials were required to allow experimental examination of both the equivalent

<table>
<thead>
<tr>
<th>Material</th>
<th>Dielectric model</th>
<th>( \varepsilon_1 )</th>
<th>( \varepsilon_\infty )</th>
<th>( f_R ) (GHz)</th>
<th>( \alpha )</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deionised water</td>
<td>Cole-Cole</td>
<td>78.3</td>
<td>4.6</td>
<td>19.7</td>
<td>0.014</td>
<td>Schwan et al (1976)</td>
</tr>
<tr>
<td></td>
<td>Debye</td>
<td>78.36</td>
<td>5.16</td>
<td>19.24</td>
<td>0.0</td>
<td>Kaatze and Uhlendorf (1981)</td>
</tr>
<tr>
<td>Methanol</td>
<td>Cole-Cole</td>
<td>33.7</td>
<td>4.4</td>
<td>3.22</td>
<td>0.036</td>
<td>Jordan et al (1978)</td>
</tr>
<tr>
<td>Ethanol</td>
<td>Debye</td>
<td>24.4</td>
<td>4.8</td>
<td>1.14</td>
<td>0.0</td>
<td>Fellner-Feldegg 1969</td>
</tr>
<tr>
<td>Propan-1-ol</td>
<td>Debye</td>
<td>20.6</td>
<td>4.2</td>
<td>0.52</td>
<td>0.0</td>
<td>Garg and Smyth (1965)</td>
</tr>
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<td>Fellner-Feldegg 1969</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Suggett et al (1970)</td>
</tr>
</tbody>
</table>
Figure 5. Deionised water reflection measurements showing the effects of resonances in the container. A, glass beaker in air; B, glass beaker in saturated saline solution; C, thin polymeric vessel in saline. +, 900 MHz; ×, 2.15 GHz.

circuit model and the point-matching technique. Non-polar materials, such as cyclohexane, have been characterised to a high degree of accuracy (Afsar et al 1980) and therefore represent excellent dielectric reference materials but their permittivities and losses are low and other standards are also required for the present work. Polar liquids provide a more suitable range of permittivities but a comprehensive literature survey revealed the sparser and less consistent nature of the polar liquid dielectric data. Of the possible materials, distilled water and the simple alcohols are best known; their dielectric parameters at 25°C in one case (methanol) were derived by interpolation equations to derive the complex permittivity reference data. Of the polar material reference data, greater confidence is probably justified for water (Kaatze and Uhlendorf 1981) than for the other liquids whose dielectric parameters at 25°C displayed significant inconsistencies within the literature.

The dielectric reference liquids were measured initially in the following manner. Approximately 200 ml of each liquid, placed in a glass beaker, was maintained in a thermostatted water bath at 25 ± 0.2°C. Reflection coefficients in the frequency range 50 MHz–2.0 GHz were obtained from the HP 8510 ANA following immersion of the sensor beneath the surface of the liquid (avoiding bubbles at the interface by inspection using a mirror). Display of the reflection coefficient data revealed the presence of artefacts at some of the higher frequencies. These were attributed to resonances set up within the beaker by internal reflection of propagating radiation. Performing the measurements in the beaker suspended in air worsened these artefacts because of an exacerbated impedence mismatch at the glass walls. The problem was reduced significantly by placing the sample in a thin-walled vessel of irregular shape (to scatter the radiation) surrounded by a dissipative medium such as saline (to absorb much of the scattered radiation). Reliable data were then achieved for all the reference materials between 50 MHz–2.0 GHz. The resonance phenomenon was worst for deionised water which is not particularly lossy at these frequencies. The main features of the artefacts are illustrated in the Smith Charts of deionised water shown in figure 5.

5. Measurement theory

5.1. Equivalent circuit model

The reflection coefficients of the six reference materials were plotted at each frequency according to equation (10). For the frequency range 50 MHz–200 MHz inclusive, it was possible to plot lines representing least-squares fits of all the data, but at higher frequencies linear fits could be achieved for successively fewer materials. For example, the datum point for water departed from the line fitting the remaining materials at 300 MHz. By 1 GHz both the water and methanol data-points deviated significantly from a best-fit line of the remaining materials. Figures 6(a) and (b) illustrate these observations. Measurements of the gradients and ordinate intersections at frequencies from 50 MHz–200 MHz enabled average values of \( C_i \) and \( C_r \) to be determined as 0.0939 ± 0.0011 pF and 0.009 ± 0.012 pF, respectively, where the uncertainties are to two standard deviations.

We attempted initially to explain these observed non-linearities on the basis of a sample-dependent increase of \( C_r \). Thus, for a homogeneous coaxial line open to semi-infinite space (in practice, a coaxial airline with a ground plane) equations have been derived (Marcuvitz 1965) and solved numerically (Gajda 1979) to show that for a 14 mm diameter sensor open to air a static value of fringing capacitance prevails until about 1 GHz. Above this frequency the principal TEM mode becomes supplemented by evanescent TM modes excited at the junction discontinuity (Bianco et al 1980). An expression of the type

\[
C(f) = C_s + AF^2
\]  

(34)

where \( C_s \) represents the static capacitance and \( A \) is a constant dependent on the sensor dimensions, has been suggested to take account of the frequency dependence of a sensor's fringing capacitance (Athey et al 1982). It is obvious, however, that this equation is inadequate to describe the UHF and MW non-linearity noted above (figure 6(b)) where the sensor sample fringing field capacitance becomes a function of both frequency and permittivity. However, the frequency promoting a fringing capacitance of a given value for a sensor open to a dielectric will be lower than that frequency promoting the same capacitance value for the sensor open to air because of the contracted wavelength of electromagnetic radiation in matter.

For a plane wave,

\[
f_o(e^*) = \frac{c}{\lambda_o} \left( \frac{1}{2} \left[ 1 + (e^* + e^{'*})^{1/2} \right] ^2 + 1 \right)^{1/2}
\]  

(35)

where \( f_o(e^*) \) is that frequency for which the radiation in a dielectric has effectively the same wavelength, \( \lambda_o \), as the...
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Figure 6. Least-squares fit of the lumped circuit fringing capacitance (equation (10)) to the permittivities of the dielectric reference materials, 200 MHz. (b) As (a) but for 1 GHz. The straight line is a least squares fit to the points corresponding to the four lower permittivity materials only.

radiation in air; this will be termed the equivalent frequency of the sensor. For high-permittivity materials, in the present formalism, equation (34) thus becomes

$$C(f, \varepsilon^*) = C_i + A f^2(\varepsilon^*).$$  

Letting

$$\Delta C(f, \varepsilon^*) = C_i(f, \varepsilon) - C_i,$$

equation (36) has been examined in the form

$$\Delta C_i(f, \varepsilon^*) = A f^2(\varepsilon^*)$$

for those frequencies and for those materials that departed from the linear equivalent circuit model: $C_i(f, \varepsilon^*)$ was derived using

$$C_i(f, \varepsilon^*) = \frac{-2|\Gamma| \sin \varphi}{\omega \varepsilon \varepsilon^* \left[ 1 + 2|\Gamma| \cos \varphi + |\Gamma|^2 \right]} - \frac{C_i}{\varepsilon^*}$$

and $f_0(\varepsilon^*)$ was calculated from equation (35) using the known reference material complex permittivities. Figure 7 presents a least-squares best-fit of this data up to 1.3 GHz. Regression analysis yielded the parameters of equation (38) as $A = 1.21 \times 10^{-4}$ and $n = 2.03$, for $f_0(\varepsilon^*)$ expressed in GHz, with a regression coefficient of 0.940. The power term is thus not dissimilar to that (2.0) suggested by Athey et al. (1982) as appropriate for describing the frequency-dependent increase in $C_i$ due to evanescent TM modes, for smaller coaxial lines open to air. A much poorer correlation, however, was found when including data obtained at frequencies above 1.3 GHz.

Figure 7. The change in the sensor fringing field capacitance as a function of equivalent frequency, $f_0$ (equation (35)). Note that the actual upper frequency limit in these measurements was 1.3 GHz.

The correlation that exists between the reference material data and equation (36) suggests an iterative method to determine the sensor capacitance appropriate for unknown materials. Initial values of $\varepsilon_i$ and $\varepsilon_i'$ may be obtained from equations (3) and (4) and applied to equations (35) and (38) to derive a new capacitance value, $C_i(f, \varepsilon^*)$. Application of this improved estimate of sample fringing capacitance yields $\varepsilon_i$ and $\varepsilon_i'$. The iteration proceeds until

$$|C_i(f, \varepsilon^*) - C_i(f, \varepsilon_{i-1}^*)|$$

becomes less than an arbitrary value, taken here as 0.0011 pF. The results of this procedure for methanol and water are shown in figure 8; the uncertainties in $\varepsilon_i'$ and $\varepsilon_i''$ were estimated using equations (7) and (8) with $\Delta C_i = 0.0011$ pF, $\Delta Z_0 = 0.1 \Omega$, $\Delta |\Gamma| = 0.007$, and $\Delta \varphi = 0.7^\circ$. For methanol, the calculated values of $\varepsilon_i'$ agree with the known reference data to within the expected uncertainties to about 1.2 GHz. The discrepancies between the calculated and reference $\varepsilon_i'$ values deviate by values larger than expected at frequencies above 800 MHz. For water, the discrepancies between the calculated and reference $\varepsilon_i'$ values fall within the expected uncertainties until 1.3 GHz; those in $\varepsilon_i''$ increase rapidly at frequencies beyond about 500 MHz. The launching of radiation by the sensor has been noted above (figure 5) and it is primarily responsible for these large discrepancies in $\varepsilon_i''$.

The three criteria cited in §2.1 as being necessary for the validity of the equivalent circuit model thus generally do not hold, and it is inadequate at some frequencies for deriving the sample complex permittivity to within a calculable uncertainty (even for those of the reference materials used in the initial parameterisation). In particular, the results shown in figure 6 indicate that $C_i$ is a function of sample permittivity and frequency, and the results in figure 8 suggest that our semiempirical correction is successful for methanol to
800 MHz and for water to 500 MHz. Other attempts have been made to improve the equivalent circuit approach. For example, Stuchly et al. (1982b) seek to correct for radiation propagation by a method that utilises the antenna modelling theorem of Deschamps (1962) and the TEM variational equations of Marcuvitz (1965). These latter equations are, however, strictly applicable only to the homogeneous case of a coaxial line with the same dielectric medium inside and outside; errors quickly increase for a lossy, high-permittivity terminating dielectric (Mosig et al. 1981). Little confidence can thus be attached to such a procedure and a more rigorous treatment is evidently required.

5.2. Sensor discontinuity field point-matching method

The point-matching method was used first to predict the sensor reflection coefficients for the dielectric reference materials and frequencies of the present study. The extent of agreement for deionised water can be seen in figure 9 where one set of measurement data has been plotted. It is apparent from figure 9(a) that $|\Gamma|$ follows the predicted curve well; residual discrepancies having an RMS deviation of 0.005 and a peak value of 0.011. The nature of these deviations points to a combination of ANA calibration errors and residual standing waves as their cause rather than to inadequacies in the PM theoretical treatment. Another source of error, which would have most effect at the lowest frequencies, is the residual conductivity of the deionised water. With a water conductivity of less than 5 mS m$^{-1}$ (estimated by the manufacturers of the water deionisation equipment used here) the reduction of $|\Gamma|$ caused by conduction could have been more than 0.005 at 100 MHz, reducing in inverse proportion to frequency. Figures 9(b) and (c) show that the discrepancy in phase amounts to no more than 0.7° for a correctly adjusted probe geometry. Residual standing waves and ANA calibration errors are again the principal sources of these discrepancies. Investigation of the effects of misalignment of individual parts of the probe tip (the inner and outer conductors and the PTFE bead) has emphasised the importance of ensuring that the probe geometry actually corresponds to the coplanar condition assumed by the PM theory. Measurements in which the probe tip was deliberately misaligned (e.g. figure 9(c)) indicate that a discrepancy of only a few micrometers can cause the total fringing capacitance to change by 1% (1 fF) or more from the calculated value. (This behaviour is not surprising when the difference in permittivity between the bead and the deionised water is considered.) It should also be pointed out that the attachment of the flange to the sensor adds 15 fF to its total capacitance in air. The fact that the sensor flange does not extend beyond 50 mm diameter must reduce the total capacitance to some extent, but this is unlikely to amount to more than 1 fF. In the event, the agreement between the measured data and the PM theory, as exemplified in figure 9,
has been shown to be within the estimated combined uncertainties. This favourable agreement between the theoretical and experimental measurements encouraged us to derive a means of achieving the inverse solution (\( \Gamma^x \rightarrow \varepsilon^x \)) as discussed later.

In order to make a direct comparison between the equivalent circuit method and the point-matching method of Mosig et al. (1981), we have computed by the inverse \( \psi \) method the total complex fringing field capacitance, \( C_\psi \), of the sensor, as defined by equation (6), for various frequencies and for various values of sample complex permittivity. We present the results normalised to the sample complex permittivity; i.e. as

\[
C_\psi \varepsilon^x = C_r - j C_i. \tag{40}
\]

In terms of the limited equivalent circuit description (equation (6)),

\[
C_r = C_i + [\varepsilon' C_i (\varepsilon'^2 + \varepsilon''^2)] \tag{41}
\]

and

\[
C_i = -\varepsilon'' C_i (\varepsilon'^2 + \varepsilon''^2). \tag{42}
\]

The computed real and imaginary capacitances, \( C_r \) and \( C_i \), are plotted in figures 10–12 against the relative permittivity of the dielectric reference materials at various frequencies between 10 MHz–2.5 GHz and for three values of loss tangent. Fringing capacitance ordinates normalised for the sensor radii are also presented on the right-hand axes for general convenience in figures 10–12. At low frequencies, where radiation effects are negligible (10 MHz), and for \( \varepsilon' = 1, \varepsilon'' = 0 \), it may be noted from equation (41) and figure 10(a) that

\[
C_r = C_i + C_t = 0.108 \, \text{pF}
\]

for our probe. At 10 MHz, for \( \varepsilon' \rightarrow \infty, \varepsilon'' = 0 \), \( C_r = C_t = 0.094 \, \text{pF} \). Therefore, \( C_i = 0.014 \, \text{pF} \). These values are within the bounds of our previous graphical estimates (§5.1). To allow for higher-frequency radiation effects, the equivalent circuit model (figure 2(b)) requires a finite parallel conductance, \( G \). Equation (42) then becomes

\[
C_i = \frac{(G \varepsilon' / \sigma) - \varepsilon'' C_t)}{(\varepsilon'^2 + \varepsilon''^2)} \tag{43}
\]

For a zero loss-tangent, \( C_i \) is thus entirely positive, while for a finite loss-tangent, a finite \( C_i \) can generate negative values of \( C_i \) as our \( \psi \) analysis confirms in figures 11(b) and 12(b). The 10 MHz (low frequency) and zero loss-tangent normalised capacitances are very similar to values previously derived (Gajda and Stuchly 1983) from an algorithm based on the method of moments, but neither the effects of the radiation nor the effects of a finite loss-tangent on the fringing field capacitance were considered in that paper. From figures 10–12, we note that \( C_r \) and \( C_i \) become functions of both frequency and \( \tan \delta \) having features not predicted from the equivalent circuit theory.

The data of figure 10(a) for \( \tan \delta = 0.0 \) is presented in a summarised fashion in figure 13 where the abscissa now represents \( f \varepsilon' \) (with \( f \) in GHz). From this presentation it can be seen that the high-frequency/high-permittivity capacitances follow a common curve which is determined by the wavelength in the sample (proportional to the square root of the abscissa). The curves at lower frequencies and permittivities, however, which represent the range of most practical interest, are still markedly frequency dependent.

Figure 14, like figure 9, compares a set of measurements on deionised water against the \( \psi \) theory. While it thus contains similar information to figure 9, its presentation in terms of permittivity-normalised fringing capacitance, \( C_\psi \), shows clearly that the concept of a fixed value for this capacitance is very far from representing the behaviour of the real fringing fields. The value of \( C_\psi \), in fact, falls quite markedly at higher frequencies in a manner which is not predicted by equation (41). At the same time the measurement uncertainty rises quite dramatically and one would not expect the 14 mm probe to give information of any value much above 2.5 GHz. Figures 10–14 clearly show that the fringing field capacitance is a complicated function of frequency and complex permittivity and illustrate why the equivalent circuit theory generally provides an inadequate physical description of the sensor/sample interface.

5.3. The inverse point-matching problem

For a known value of \( \varepsilon^* \) (and a given frequency) we may compute the reflection coefficient \( \Gamma^x(\varepsilon^*) = R_0(\varepsilon^*) \) as described in §2.2. Conversely, therefore, given the reflection
coefficient $\Gamma^*$ (for a known frequency) we may compute the corresponding permittivity $\varepsilon^*$ as a zero of the function

$$F(\varepsilon^*) = R_0(\varepsilon^*) - \Gamma^*.$$  \hspace{1cm} (44)

We do this by an iterative technique (of Newton–Raphson form) based on the first-order approximation

$$F(\varepsilon^*) = F(\varepsilon', \varepsilon'') = F(\varepsilon', \varepsilon'') + (\delta F / \delta \varepsilon') \Delta \varepsilon' + (\delta F / \delta \varepsilon'') \Delta \varepsilon''$$ \hspace{1cm} (45)

to $F(\varepsilon^*)$ near $(\varepsilon', \varepsilon'')$. If $(\varepsilon', \varepsilon'')$ is an approximation to the zero of $F$, then, setting $F(\varepsilon^*) = 0$ and equating real and imaginary parts, equation (45) yields two linear equations for the increments $\Delta \varepsilon'$ and $\Delta \varepsilon''$. The solution of these equations yields a revised approximation

$$\varepsilon^*_{i+1} = (\varepsilon'_{i+1}, \varepsilon''_{i+1}) = \varepsilon'_{i+1} - j \varepsilon''_{i+1}$$ \hspace{1cm} (46)

to $\varepsilon^*$, where

$$\varepsilon'_{i+1} = \varepsilon'_{i} + \Delta \varepsilon'_{i}, \quad \varepsilon''_{i+1} = \varepsilon''_{i} + \Delta \varepsilon''_{i}$$ \hspace{1cm} (47)

Starting from an initial estimate $\varepsilon^*= (1.0, 0.0)$, the iterative scheme defined here converges fairly rapidly in most of the cases considered so far. Only in exceptional circumstances do the iterates begin to diverge and then $\varepsilon''$ becomes negative for some $i$. When this happens, we simply reset $\varepsilon''$ to zero and continue the iteration which then converges.

6. Measurements

We present here only a small sample of the measurements which have been carried out in order to demonstrate the power of the PM technique. Several alcohols were measured by the calibrated dielectric sensor in the manner described in §4.2. The inverse PM procedure was applied to the measured...
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data; the complex permittivities calculated for propan-1-ol, ethanol and glycerol are shown in the form of Cole–Cole plots in figure 15. The uncertainties were estimated from our analyses to follow approximately a linear increase of $\Delta \varepsilon$ and $\Delta \omega$ from 0.003 and 0.3° at 50 MHz to 0.007 and 0.7° at 1 GHz and above, respectively. The data for ethanol and propan-1-ol were analysed by a least-squares minimisation program, DEBYEFIT, to compute Debye relaxation parameters which can be compared with the previously published data in table 1. The parameters obtained for 25 °C were

ethanol:

$$\varepsilon_\infty = 24.50 \quad \varepsilon_1 = 4.52 \quad f_R(\text{GHz}) = 0.962 \text{ GHz}$$

propan-1-ol:

$$\varepsilon_\infty = 20.44 \quad \varepsilon_1 = 3.80 \quad f_R(\text{GHz}) = 0.510 \text{ GHz}.$$ 

Recent theories explain the suitability of the Debye theory for simple alcohol dielectric relaxations by invoking a collective polarisation of molecular clusters in solution (Minami et al 1981, Levin and Feldman 1982). A fit of the Debye function to the present glycerol data was not attempted since it does not conform to this Debye behaviour. Tables of all the measured data are included in a separate report, as are data on biological tissue measurements (Grant et al 1988).

7. Discussion and conclusions

From the analyses presented in §§5.1 and 5.2 it is apparent that as a means of relating $T$ directly to $\varepsilon^* \text{ existing equivalent-circuit models of the fringing fields of a dielectric sensor fail to cover adequately the wide range of frequencies and sample complex permittivities for which the sensor would otherwise
be perfectly suitable. The results of figures 10–14 show that the total fringe capacitance, $C_F$, is a complicated function of frequency, sample permittivity and loss and any theory that assumes a linear dependence of fringe-field capacitance with permittivity or which ignores radiative effects can clearly only be approximate, and will generally fail badly at higher frequencies. Furthermore, it could be difficult to derive calculable uncertainties and corrections for equivalent-circuit models if the sensor does not possess a geometry amenable to a suitable numerical analysis (such as the PM analysis). These conclusions are of relevance to any critical assessment of the relatively large body of dielectric data that has been obtained from polar and biological materials by equivalent circuit methods (see e.g. Stuchly et al. 1982a). We would anticipate that they would also apply to related geometries such as those discussed by Burdette et al. (1977), Toler and Seals (1977) and Burdette et al. (1980) who present results of the $RF$ and $SW$ complex permittivities of lossy materials measured by small ($b < 1.1$ mm) monopole probes (similar to figure 1 with the inner conductor extended by a short length).

Of necessity, many workers have recognised the limitations of the model described by figure 2(b). The relevant equations given by Tai in Jasik (1961) strictly hold only for geometries with one dielectric species present and in the absence of radiation. Burdette et al. (1980) utilise second-order terms (similar to those suggested by equation (34)) to correct capacitance and loss. More recently, Gabriel et al. (1986) have employed time-domain spectroscopy for dielectric measurements with an open circuit sensor, similar to figure 1, but with a coaxial outer diameter of 7 mm. They have also allowed for similar corrections for loss and capacitance change with frequency. However, all of these corrections must suffer some element of approximation and in some cases they employ free parameters which require empirical fitting to measured data for the model to work at all. We would hope that our work could provide a basis upon which the effects of these approximations could be judged.

The results presented in figures 10–14 indicate that the discrepancies of 10% or more occur between sensor terminating admittances computed from simple models at 1 GHz and those computed by the PM analysis. Discrepancies in conductance for relatively low-loss materials like deionised water can be much higher. The discrepancies increase at higher frequencies. It is important to ask whether such discrepancies would necessarily have propagated into the derived complex permittivities of other workers who have employed approximate equivalent circuits. There are, in fact, two reasons why their effects could be reduced. Firstly, the practice of calibration of sensors by means of reference liquids could partially correct for changes in $C_i$ and $C_r$ provided the reference liquids had similar properties to the dielectric under test (as suggested by Gabriel et al. 1986) for then similar values of $C_i$ and $C_r$ would apply to both reference liquid and specimen and a form of compensation would occur during the calibration procedure. However, significant compensation could only be expected if one of the reference liquids had very similar properties to the specimen and it is unlikely that one would obtain such references for all possible specimens at all frequencies. Secondly, a referee has pointed out to us that we have employed a larger sensor above 1 GHz than most other workers and that radiative effects (and hence changes in $C_i$ and $C_r$) would be less for smaller sensors at a given frequency. This is true, but the magnitude of such effects must be expected to scale in frequency inversely with the linear dimensions of the sensor so that where we find it necessary to take radiation into account typically above 600 MHz, a user

of a 3.5 mm diameter sensor, say, should certainly take similar precautions above 2.5 GHz.

One important result of this work has been the close agreement between the PM-predicted and measured reflection coefficients of polar materials over a wide frequency range (e.g. figures 9, 14). Our work thus represents an independent experimental check of the PM theory of Mosig et al. (1981). This corroboration of the PM theory has also enabled us to derive an inverse solution, to apply it to the determination of alcohol complex permittivities and to estimate associated measurement uncertainties (figure 15). In the longer term, though, it will still be desirable to check the PM theory by an independent numerical analysis because it is difficult to estimate the computational errors without some form of intercomparison.

It is clear that the use of a calculable open-ended sensor with all ANAS such as the HP 8510 and the application of the inverse PM theory provide a rigorous approach to non-invasive dielectric measurements for which the measurement uncertainties can be realistically estimated. The principal drawback of this approach, however, is the need for a powerful computer to process the lengthy iterative calculations. Furthermore, ANAS are instruments which may, for many purposes, be prohibitively expensive. Cost and efficiency considerations for non-intrusive measurements also suggest the eventual use of a small portable reflectometer with the sensor. The six-port reflectometer, possibly employing diode detection, could provide such an instrument, as investigated by Zurcher et al. (1983). Future work should also be directed towards simplifying the mathematical analysis and implementing the equations on a dedicated computer. A simplified analysis based on polynomial approximations has, in fact, been reported (Sphicopoulos et al. 1985) and this may prove well suited for implementation on a microcomputer. Alternatively, absolute calculability may not be required if the measurements are taken over a limited range of frequencies and permittivities. Semiempirical modelling using the equivalent circuit approach might then be adequate, and results such as those given in figures 10–12 should prove useful for parameterising the approximate theory. The provision of suitable reference liquids for probe calibrations would continue to be an important requirement for this application.

We have employed a dielectric sensor with a fringing field capacitance substantially greater than has previously been reported. The larger fringing capacitance may prove useful for bio-dielectric measurements at frequencies lower than those of this and most previous work. Of particular interest would be investigations of the predicted maximum permittivity differences between normal and malignant tissues in the frequency range 1–10 MHz (Grant and Spyrou 1985). Further improvements in the design of a specifically calculable dielectric sensor would require consideration of the coplanarity of the outer and inner conductors and of the dielectric support medium (the required tolerance is ideally less than 1 μm), use of sharp edges on all metallic and dielectric parts and the investigation of the effect of the finite ground plane. A 'thick' dielectric bead support is required. If it is too thin, evanescent TM modes launched or reflected at its surface will interact and render the point-matching theory invalid. The required thickness can be calculated from the propagation constants of the TM modes which emerge from the point-matching theory, equation (11). In general, any particular application of the sensor, e.g. biomedical tissue characterisation or industrial quality control, would require selection of optimum measurement frequencies and probe capacitances (Grant 1984).

We have shown that resonances caused by total internal reflections within a dielectric specimen can lead to significant
errors in measuring low and medium loss high-permittivity materials (figure 5). We are not aware of this problem having been addressed in the literature before but have shown that it can be reduced to small proportions (1% or less in |E|²) by carefully matching the specimen to more lossy surroundings. It is unlikely that this effect is significant in biomedical applications where losses are generally high (tan δ > 0.3). However, post-calibration of a calculable sensor (to allow for geometrical imperfections) and any semiempirical standardization of a non-calculable sensor both require the use of reference liquids. As far as we are aware, the only such liquid that has been adequately characterised to a 1% level for use as a polar standard is deionised water. Its loss is sufficiently low below 2.0 GHz (tan δ < 0.1), however, for the resonance problem to be significant. Saline solutions of different molarities should provide useful lossy dielectric reference materials in which the resonance problem would be much reduced. Indeed, Stogryn (1971) has provided empirically based formulae for saline complex permittivities as functions of temperature and molarity to allow these solutions to be used in just this way, but the data upon which his formulae are based were obtained in the 1940s–1960s. There is clearly a requirement for such an exercise to be repeated using more recent measurement techniques.

To summarise briefly the findings of this work, we can say that sensor calculability has enabled us to identify and quantify sources of uncertainty which are important even for non-calculable sensors. We have demonstrated agreement within 0.007 in |E|² and 0.7° in phase between theory and measurement for deionised water in the frequency range for which our sensor was designed. We have developed an inverse solution of the point-matching theory and have used it for complex permittivity measurements of alcoholic solutions in the frequency range 50 MHz–2.0 GHz. In vivo measurements of human skin were also performed and these results have been published elsewhere (Grant et al. 1988). We feel that the work has clarified the basic principles of the sensor technique, pointed to design considerations for improved sensors, and led to new applications of the technique for industrial quality control and biomedical research.

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