Leaky Mode Dispersion in Omega NRD Waveguides

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Abstract — **This paper addresses the material dispersion effects on the propagation characteristics of an omega nonradiative dielectric (NRD) waveguide. The omega NRD waveguide is an H-guide, working in the closed-waveguide regime, where the common isotropic slab is replaced by an omega slab. For the first time, a single-resonance lossless model is used for the material constitutive parameters of the omega NRD waveguide. The dispersion diagrams reflect the dispersive characteristics of the medium. In addition to the surface guided modes, proper leaky modes are found to propagate in the frequency range above the resonance, where some of the constitutive parameters become negative.**

I. INTRODUCTION

The inherently dispersive characteristics of an omega medium, affects its application to microwave and millimeter wave circuits. The resonant behavior of the inclusions causes the medium to be dispersive, even in small frequency bands. Above the resonant frequency, an omega medium can behave like a double negative (DNG) medium, exhibiting simultaneously negative permittivity and permeability [1]. DNG NRD waveguides can support proper leaky modes [2], due to the backward characteristics of the DNG metamaterial.

On the other hand, proper leaky modes are also known to propagate in pseudochiral omega slabs [3], even when all constitutive parameters are positive. This paper addresses, for the first time, the propagation of proper leaky modes in a dispersive omega NRD waveguide.

The omega nonradiative dielectric (NRD) waveguide is an omega H-guide operating as a closed-waveguide to prevent lateral radiation [4, 5]. In this paper, the propagation of proper leaky modes in an omega NRD waveguide is investigated, assuming a lossless dispersive model for the constitutive parameters.

The omega NRD waveguide [6] consists of an omega slab, sandwiched between two metal plates (Fig. 1). If the spacing is less than half of the free-space wavelength, lateral radiation from the structure is prevented, since higher-order parallel-plate waveguide modes are below cutoff, and the TEM parallel-plate waveguide mode is not excited provided that symmetry with respect to the y axis is maintained. In both the frequency ranges considered in this paper, the H-guide is operating in the closed-waveguide regime.

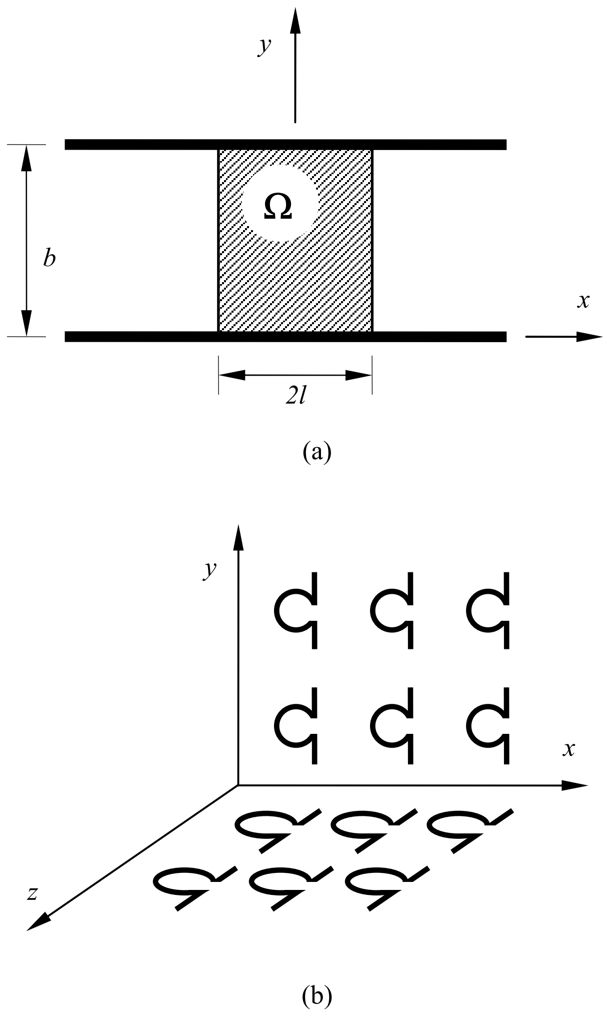


Fig. 1 Geometry of the omega NRD Waveguide: a) Cross section with geometrical parameters and coordinate system; b) Spatial orientation of the omega particles inside the isotropic host medium.

II. MATERIAL DISPERSION

Artificial omega composite materials are made of omega shaped inclusions distributed inside a host medium. Omega particles are planar inclusions which can be seen as a combination of a wire and a loop antenna. When an external electrical field is applied to an omega particle, it induces an electrical dipole moment parallel to the wire and a magnetic moment orthogonal to the loop. Therefore, omega media are magneto-electric media.

The uniaxial omega medium, as depicted in Fig. 2, can be characterized by the following constitutive relations

$$\begin{cases} \mathbf{D} = \varepsilon_0\, \boldsymbol{\varepsilon} \cdot \mathbf{E} + \sqrt{\varepsilon_0 \mu_0}\, \boldsymbol{\xi} \cdot \mathbf{H} \\ \mathbf{B} = \sqrt{\varepsilon_0 \mu_0}\, \boldsymbol{\zeta} \cdot \mathbf{E} + \mu_0\, \boldsymbol{\mu} \cdot \mathbf{H} \end{cases} \quad (1)$$

with $\boldsymbol{\varepsilon} = \varepsilon_{\parallel}\hat{\mathbf{x}}\hat{\mathbf{x}} + \varepsilon_{\perp}(\hat{\mathbf{y}}\hat{\mathbf{y}} + \hat{\mathbf{z}}\hat{\mathbf{z}})$, $\boldsymbol{\mu} = \mu_{\parallel}\hat{\mathbf{x}}\hat{\mathbf{x}} + \mu_{\perp}(\hat{\mathbf{y}}\hat{\mathbf{y}} + \hat{\mathbf{z}}\hat{\mathbf{z}})$ and $\boldsymbol{\xi} = \boldsymbol{\zeta} = j\Omega(\hat{\mathbf{y}}\hat{\mathbf{z}} - \hat{\mathbf{z}}\hat{\mathbf{y}})$.

The resonant behavior of the omega inclusions will cause the medium to be dispersive. Therefore, it is necessary to consider a dispersive model for the constitutive parameters, even if the analysis is reduced to a small frequency band. The model herein adopted was first proposed in [6]. According to this model, losses as well as mutual impedance between inclusions are neglected. Moreover, there will be a frequency dependence only along the y and z directions, where magneto-electric coupling occurs. For a given host medium, the model depends on three parameters: (i) the resonance frequency f_r; (ii) the variation $\Delta_{\varepsilon_\perp}$ of $\varepsilon_\perp$ caused by the inclusions at low frequencies; (iii) the variation $\Delta_{\mu_\perp}$ of $\mu_\perp$ caused by the inclusions at high frequencies, according to

$$\varepsilon_{\perp} = \varepsilon_{\perp}^{h} + \frac{\Delta\varepsilon_{\perp}}{1 - \left(\frac{f}{f_r}\right)^2} \quad (2a)$$

$$\mu_{\perp} = \mu_{\perp}^{h} + \frac{\left(\frac{f}{f_r}\right)^2}{1 - \left(\frac{f}{f_r}\right)^2}\Delta\mu_{\perp} \quad (2b)$$

$$\Omega = \frac{\frac{f}{f_r}}{1 - \left(\frac{f}{f_r}\right)^2}\sqrt{\Delta\varepsilon_{\perp}\Delta\mu_{\perp}} \quad (2c)$$

In all the simulations presented, the following values have been adopted: $f_r = 110\,\mathrm{GHz}$ $\varepsilon_{\parallel}^{h} = \varepsilon_{\perp}^{h} = 2.0$, $\mu_{\parallel}^{h} = \mu_{\perp}^{h} = 1.0$, and $\Delta\varepsilon_{\perp} = \Delta\mu_{\perp} = 1.0$. The variation of the dispersive parameters in (2) is depicted in Fig. 2. While all parameters are positive below the resonance, some become negative above the resonance.

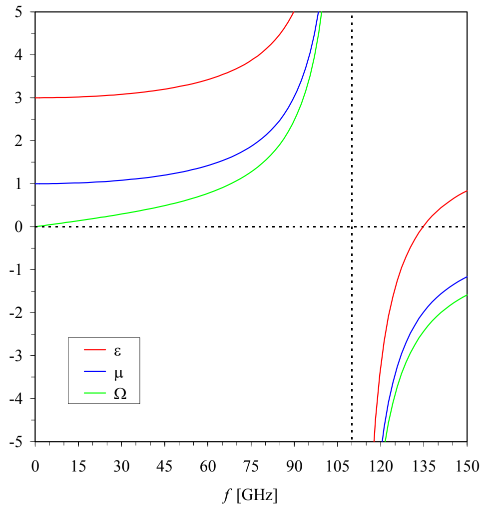


Fig. 2 Material dispersion of the omega medium parameters in the vicinity of the resonance frequency. Before the resonance, all the parameters are positive while, after the resonance, some parameters can be negative.

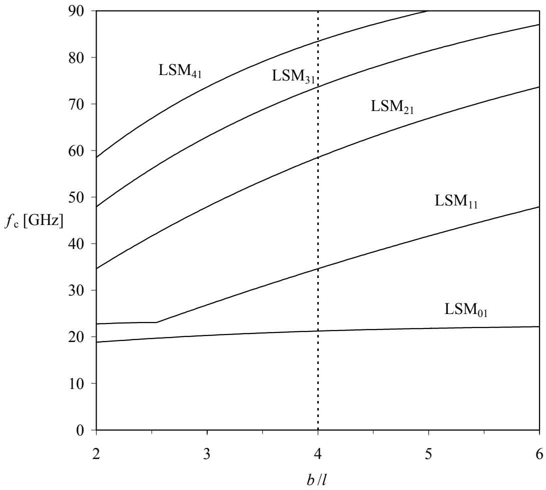
III. The Uniaxial Omega NRD Waveguide

The dispersionless uniaxial omega NRD waveguide was already analyzed by the authors in [4]. As was then proved, the discrete spectrum of an NRD waveguide filled with an uniaxial omega medium, as depicted in Fig 1, can be expressed in terms of a set of longitudinal section electric (LSE) and longitudinal section magnetic (LSM) hybrid modes. The modal equation for the LSM_{mn} modes of an omega NRD waveguide is [4]

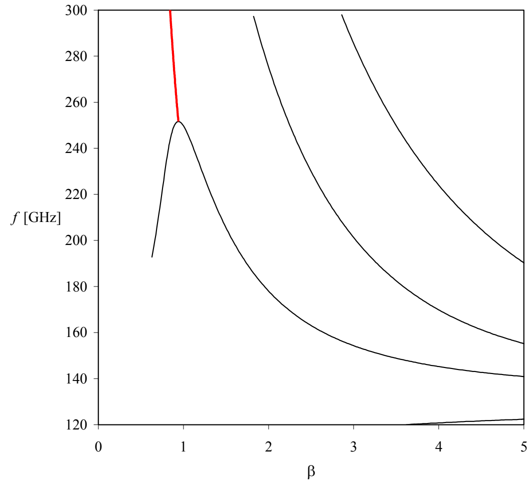
$$\left[h\cot(hl) + \alpha\varepsilon_{\perp}\right]\left[h\tan(hl) - \alpha\varepsilon_{\perp}\right] + \Omega^2 = 0 \quad (3)$$

where $h = k_0\sqrt{\varepsilon_{\perp}\mu_{\perp} - \Omega^2 - \varepsilon_{\perp}(\beta_y^2 + \beta^2)/\varepsilon_{\parallel}}$, and $\alpha = k_0\sqrt{\beta_y^2 + \beta^2 - 1}$, with $\beta_y = n\pi / b$ and n being an integer. Using the duality principle, the modal equations for the LSE modes can be easily derived directly from (3). However, due to its lower attenuation losses, the most interesting mode in this waveguide is the LSM_{01} mode. Therefore, without loss of generality, this paper is restricted to the modal analysis of LSM modes.

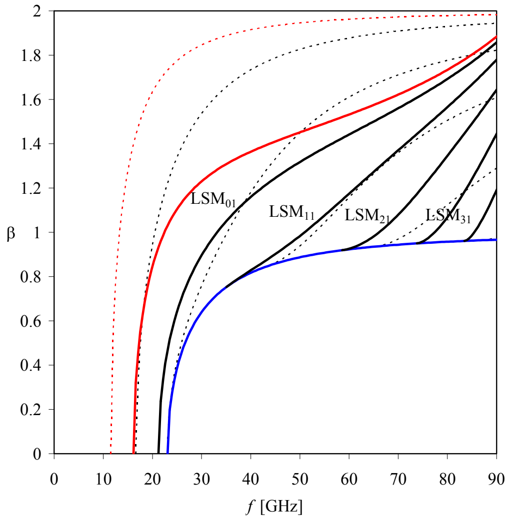
The cutoff frequency of the first LSM modes propagating in an omega NRD waveguide operating below the resonance frequency, is depicted in Fig. 3-(a) as a function of the aspect ratio b/l, for $b = 6.5\ \mathrm{mm}$. The corresponding dispersion diagram is depicted in Fig. 3-(b) for $b/l = 4.0$, where the blue line stands for $\alpha = 0$ and the red line for $h = 0$.



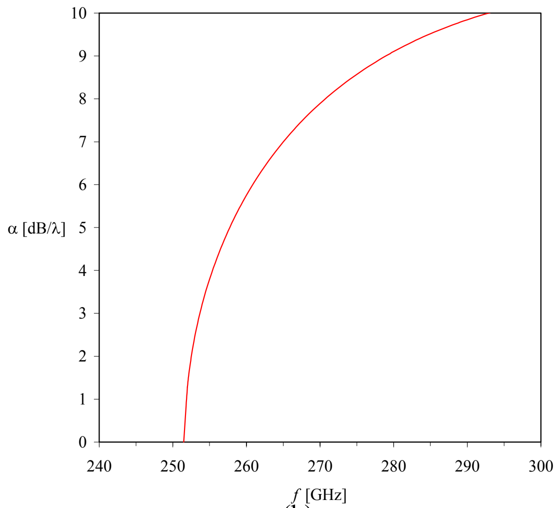
(a)



(a)



(b)



(b)

Fig. 3 First propagating LSM modes of an omega NRD waveguide with $b = 6.5$ mm, working below the resonance frequency: (a) cutoff frequency as a function of the aspect ratio b/l; (b) Dispersion diagram for $b/l = 4.0$, where the blue line stands for $\alpha = 0$ and the red line for $h = 0$, while the dashed lines are for the non-dispersive case [5].

Fig. 4 Surface and leaky LSM modes of an omega NRD waveguide with $b = 1.0$ mm and $b/l = 4.0$, operating above the resonance frequency; (a) dispersion diagram; (b) leaky constant versus frequency.

IV. Proper Leaky Modes

In this section, the waveguide is considered to be operating above the resonance where, at least, some constitutive parameters may be negative. For the leaky modes to be investigated, the longitudinal normalized wavenumber must be written as $k = \beta - j\alpha$, where β is the normalized phase constant or slow-wave factor and α is the normalized attenuation constant.

The dispersion diagram for the first propagating LSM modes of an omega NRD waveguide with $b = 1.0$ mm and $b/l = 4.0$ is presented in Fig. 4-(a). The thin black lines represent the surface modes while the thick red line represents a proper leaky mode. In this frequency range, all the depicted surface modes are super-slow modes. Moreover, the first LSM mode turns into a leaky mode at a given frequency. The variation of the leakage constant as a function of frequency is depicted in Fig. 4-(b).

V. CONCLUSION

A single-resonance lossless dispersive model was used for the material constitutive parameters of an omega NRD waveguide. The propagation of LSM modes has been analyzed in two different frequency bands: one below and another above the particle resonance frequency. Apart from the dispersive effects, no novel effects were found in the frequency band below the resonance frequency. However, in addition to the super-slow surface guided modes, proper leaky modes were found to propagate in the frequency range above the resonance, where some of the constitutive parameters become negative. This effect may suggest potential applications. Further developments on this topic will be addressed in a future publication, to include the presence of losses and the waveguide characterization near the resonance.

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