FDTD with selective update: a method to minimize the simulation time

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***Abstract* — The computational simulation of electromagnetic phenomena is important for a wide class of applications, and in the last decade had a very interesting increasing. This growth is due in part to the appearance of diverse mobile systems of communications and the development of computing industry. Yee's Finite-Difference Time-Domain (FDTD)[1-2] scheme is a well-known method for the numerical solution of such problems including the propagation of electromagnetic waves in both indoor and outdoor environment. However, the requirements of the simulation method of a given area can lead a considerable computational complexity. In this paper we compare the simulation time of the conventional method and the method with selective update. The starting point of our approach is the use of an accurate algorithm and then the development of an efficient algorithm to reduce the computation load. We make this study with a *2D* scenario with an equivalent area of 30m×15m. The results generated by both methods are compared to allow the validation of the technique.**

I. INTRODUCTION

Maxwell's partial differential equations represent a fundamental unification of electric and magnetic fields predicting electromagnetic wave phenomena. Now engineers and scientists worldwide use computers ranging from simple desktop to massively parallel arrays of processors to obtain solutions of these equations for the purpose of investigating electromagnetic wave guiding, radiation, scattering phenomena and technologies. During the two last decades, lot of software's has been developed to help telecommunication operators to develop their radio networks. The problem hardness depends on the wave frequency, the grid size and the propagation environment. With the very fast deployment of cellular phone systems, the main efforts were first dedicated to outdoor applications. In this context, the high size of environments was such that empiric methods were often preferred. Later, the fast increase of deployed radio cells in urban areas called for more specific methods taking into account a more precise definition of the cells geometry more precisely the geometry of the cells. Deterministic methods were then widely proposed, developed in a geometrical optic framework and using extensively the unified theory of diffraction (UTD), providing a wide range of methods (e.g. ray-tracing, ray launching, ray tube based algorithms) [3-5]. Usual approaches previously developed for are based either on empiric or raytracing technics. The former suffers a lake of accuracy while the later offers the possibility of a trade-off between accuracy and computation load. However, increasing the accuracy of these approaches implies the drastic increase of the computation load.

Another method, important in this area of simulation, is the FDTD (Finite Difference Time-Domain) [6], in which the electric and magnetic fields are described by vectors, both discretized in time and space. In fact, each component of the electric and magnetic field vectors describe its value in a particular cell in space, at a given time. The partial derivatives that occur in the Maxwell's equations are approximated by centered finite differences. The time stepping iterative procedure is repeated until the desired time response for the electromagnetic problem is obtained. Note that the field values need to be updated on all grid points, which are the major consumptions of computational time and memory storage. This paper describes an update to the conventional FDTD for the purpose of outdoor propagation prediction. The objective is to minimize the simulation time that is proportional to the grid dimension. In this study we make a comparison between the simulations times obtained with both methods: the FDTD conventional and with selective update, in an outdoor propagation area of 30mx15m. The obtained fields distributions with both methods are also compared to validate the technique.

II. FDTD

Finite-difference time-domain is a popular computational electrodynamics modeling technique, easy to understand and easy to implement in software. Since it is a time-domain method, solutions can cover a wide frequency range with a single simulation run. The time-dependent Maxwell's equations are discretized using central-difference approximations to the space and time partial derivatives. The resulting finite-difference equations are solved in software in a leapfrog manner: the electric field vector components in a volume of space are solved at a given instant in time; then the magnetic field vector components in the same spatial volume are solved at the next instant in time; and the process is repeated over and over again until the desired transient or steady-state electromagnetic field behavior is fully evolved.

Using FDTD, we can define a function in time and in space calculated in a generic point of the grid as:

$$F^n(i,j,k)=F(i\Delta x, j\Delta y, k\Delta z, n\Delta t) \quad (1)$$

where *i*, *j*, *k* and *n* are integers; *Δx*, *Δy*, and *Δz*, are the sizes of cells in the direction of coordinated axes; *Δt* is the time increment, made uniform across the simulation. For problems in which sources and materials have translation symmetry in, say, the *z* direction, the radiated electromagnetic field quantities will be independent of the *z* coordinate. All *z* derivative terms in Maxwell's equations become zero, and the problem effectively only has two spatial dimensions. These types of problems are commonly referred to in computational electromagnetics as *2D*. There are two independent *2D* problems, depending on the vector direction of the sources. If the sources in the problem are currents flowing in the *z* direction, then the magnetic field can only have *x* and *y* components. This situation is designated by transverse magnetic mode TM. Alternately, if the current flows in the *x*-*y* plane, the electric field intensity vector is also in the *x*-*y* plane, and the problem is transverse electric – TE.

Maxwell's equations are discretized using a Yee cell. For the TE mode, the Yee cell consists of samples of H_z on a rectangular grid. The electric field component E_x is sampled on a grid staggered by one half grid step in the *y* direction, and E_y is sampled on a grid staggered by one half grid step in the *x* direction. For a TE problem, if we solve the difference equations for the forward time sample, we obtain [2]:

$$H_z^{n+1/2}(i,j)=\left(\frac{1-\frac{\rho'_{i,j}\Delta t}{2\cdot\mu_{i,j}}}{1+\frac{\rho'_{i,j}\cdot\Delta t}{2\cdot\mu_{i,j}}}\right)H_z^{n-1/2}(i,j)+\left(\frac{\Delta t}{\mu_{i,j}}\right)\left(\frac{E_y^n(i+1,j)-E_y^n(i,j)}{\Delta x}+\frac{E_x^n(i,j+1)-E_x^n(i,j)}{\Delta y}\right) \quad (2)$$

$$E_x^{n+1}(i,j)=\left(\frac{1-\frac{\sigma_{i,j}\cdot\Delta t}{2\cdot\varepsilon_{i,j}}}{1+\frac{\sigma_{i,j}\cdot\Delta t}{2\cdot\varepsilon_{i,j}}}\right)E_x^n(i,j)+\left(\frac{\frac{\Delta t}{\varepsilon_{i,j}}}{1+\frac{\sigma_{i,j}\cdot\Delta t}{2\cdot\varepsilon_{i,j}}}\right)\left(\frac{H_z^{n+1/2}(i,j)-H_z^{n+1/2}(i-1,j)}{\Delta x}\right) \quad (3)$$

To implement the FDTD method in practice, we need to add sources, apply boundary conditions, and ensure that the solution is stable. To ensure that solution is stable the size of the cells and the incremental time is related by [2]:

$$\Delta t \le \frac{1}{c_0\cdot\sqrt{\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}}} \quad (4)$$

The dimension of a FDTD grid is dependent of the frequency since that method requires at least 10 cells for wavelength, and therefore for high frequency the grid dimension can be considerable. In this study we use the scenario presented in the Figure 2, with an area of 30m×15m, for the frequency of 1 GHz.

III. FDTD ALGORITHM COMPLEXITY

The FDTD algorithm complexity is propotional to the number of operations necessary to determine the electric and magnetic field of the overall grid.

The number of mathematical operations, i.e. the number of additions, multiplications, subtractions and divisions, is determined from Equations 2 and 3, resulting in the Equation 5 and 6.

Assuming a uniforme simulation grid and a constante simulation step, the *Δx*, *Δy*, *Δz* and *Δt* are constants, therefore four mathematical operations are necessary to calculate the electric field, i.e. two multiplications, one subtraction and one addition. Six operations are necessary to calculate the magnetic field. Given the same wieght to all operations results,

$$2\times\left(\sum_{i=1}^{N}\sum_{j=2}^{N-1}4\right)=8N^2-16N \quad (5)$$

for the electric field, and

$$\sum_{i=1}^{N-1}\sum_{j=1}^{N}6=6N^2-6N \quad (6)$$

for the magnetic field.

To calculate all the electric and magnetic fields of a NxN grid a total of *f*(N) operations are necessary on every time step.

$$f(N)=14\cdot N^2-24\cdot N \quad (7)$$

The number of operations necessary for the overall simulations can be calculated from Equation 8.

$$g(n,N)=n\cdot f(N) \quad (8)$$

Where, n is an integer that represents the number of simulation steps and *N* is the grid dimension.

From Equation 7, we conclude that FDTD algorithm's complexity is O(N^2). And Equation 8 shows that the number os CPU operations is linear for a scenario of NxN cells.

IV. FDTD WITH SELECTIVE UPDATE

The selective update method defines a region of interest (ROI) centred on the electomagnetic source, to calculate and update the electric and magnetic fields, excluding from calculation all cells outside the ROI. The dimensions of the ROI are incremented, on each simulation step, to include the eletromagnetic front wave, see Figure 1. For the selective update method become error free, the ROI area must increase at the same velocity of the eletromagnetic wave propagation.

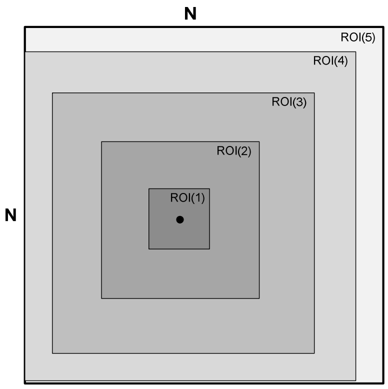

Figure 1 - Image of several region of interest for diferente simulation instant

On each simulation step the complexity of the FDTD with selective update remains the same, see Equation 8. However, with selective update the simulation initial time is reduced, because, at the beginning there are fewer cells to process. Equation 9 shows the required number of operations for n simulations steps.

$$g(n) = \sum_{i=1}^{n} f(i) \qquad (9)$$

With $i \in \mathbb{N}$ and,

$$f(i) = \begin{cases} 2+i & , i < (N-2) \\ N & , i \geq (N-2) \end{cases} \qquad (10)$$

The advantage of processing only the cells within the ROI will end when the ROI dimensions become equal to the simulation scenario, i.e NxN.

V. SIMULATION AND RESULTS

Both methods were implemented on MATLAB and tested on two diferent CPU as can be seen on Table I. The scenario used on the simulations represents an area of 30m by 15m on a 1000x500 points grid, see Figure 2. This scenario was adapted to the CPU and memory capacity and consists of several building obstacles resized. The scenario reduction was necessary because the grid size necessary for a 1:1 scale at a frequency of 5 GHz will be 50000x25000 grid points using ten cells for wavelength.

Figure 2 - Scenario used for the simulations

The electromagnetic source is located on the coodinate (626,147) and represented on the Figure 2 by a black dot. The magnetic field stregth applied is discribed by Equation 11,

$$H_{source}(n) = 100 \cdot \sin(\omega \cdot n \cdot \Delta t) \cdot gaussfunc\left(n, \frac{1500}{5}, \frac{1500}{2}\right) \qquad (11)$$

and ploted in Figure 3.

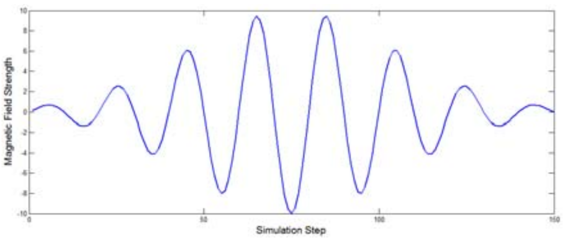

Figure 3 Source

The Table I present a performance comparison between the two methods on two diferente CPUs.

Table I Comparison of time of simulation in two CPUs

Machine	FDTD [s]	FDTD w/ Sel. Update [s]
Core2Duo T7300 2GHz and 2GB RAM	387,2	313,5
Core2Duo 6300 1,86GHz and 2GB RAM	474,7	384,7

Figure 4 shows the performance of both methods for 1500 steps of simulation with the scenario of Figure 2. As can be seen the FDTD with selective update has a time gain in comparison with the conventional FDTD method.

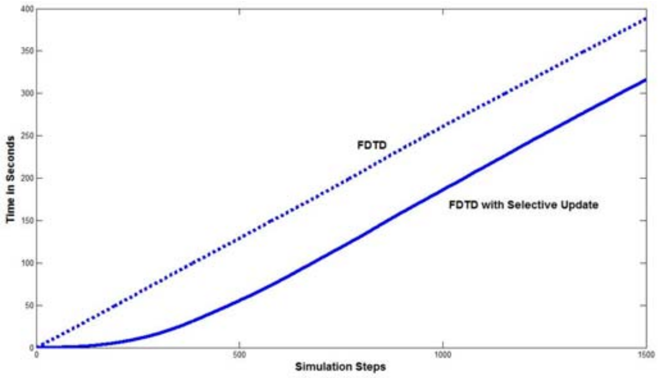

Figure 4 – Simulation time

The next figures shows the evolution of the electric field component *x*, with the time for different time steps.

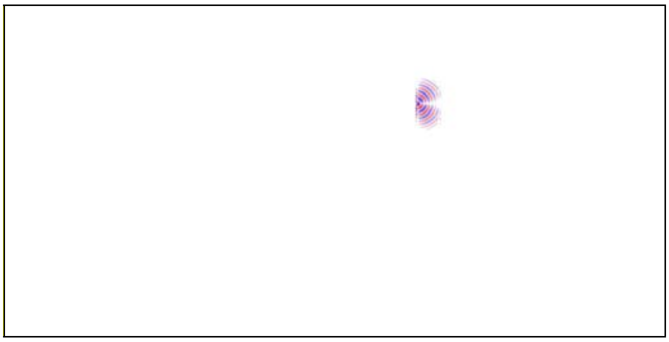

Figure 5 – E_x at simulation step 100.

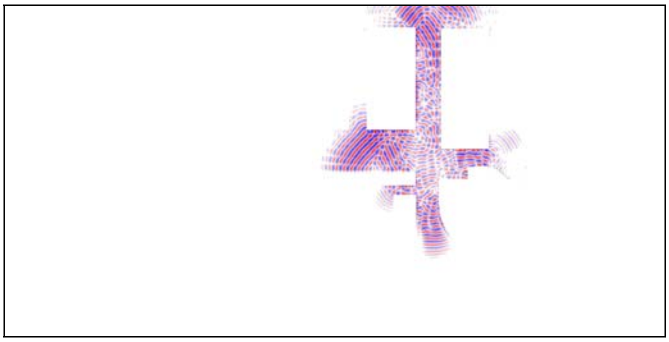

Figure 6 – E_x at simulation step 500.

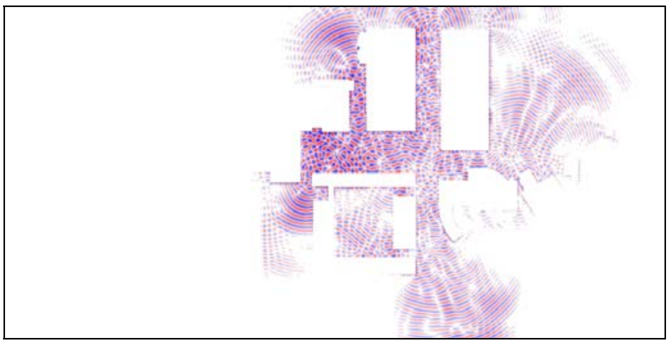

Figure 7 – E_x at simulation step 900.

VI. CONCLUSIONS

In this paper we present an update to the conventional FDTD with the objective to minimize the simulation time. With the selective update the simulation time decrease. This reduction is more pronounced at the beginning of the simulation because the ROI is small. With the increase of the ROI the simulation time enlarge and equals the FDTD conventional when the ROI dimensions become equal to the simulation scenario. These results can be observed in Figure 4. This method shows a gain at the beginning of the simulation but has the same performance when the fields reach the limit of the grid. From that moment the simulation time is identical in the two methods. However, is important to refer, that the initial gain in time is significant when compared with the global time of the simulation. The simulated results of the propagation in the scenario of Figure 2 by both methods are equal and therefore the selective update is validated.

For future work the concept/method must be extended to the idea of processing only zones where the eletromagnetic field is present and not only the zone behind wavefront. This approach increases the algorithm complexity and must be tested to confirm if an effective gain is achived. The limits of zones that must be processed and the limits of the zones that must not be processed must also be stored

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