MMSE Receivers for Non-Linearly Distorted OFDM Signals

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***Abstract*—In this paper, we have derived the structure of the minimum mean-squared error (MMSE) receiver for non-linearly distorted orthogonal frequency division multiplexer (OFDM) signals in additive white Gaussian noise (AWGN). We demonstrate that this receiver consists of a set of correlators or a set of filters matched to every possible frequency components in the non-linearly distorted OFDM signal followed by a simple linear transformation. Simulation results show that the MMSE receiver outperforms the conventional one, particularly in the presence of strong nonlinearities. Finally, we conjecture that this receiver is useful in OFDM based fiber-radio applications.**

***Index Terms*—OFDM, MMSE receiver, non-linear distortion, radio over fiber.**

I. Introduction

Orthogonal Frequency Division Multiplex (OFDM) is a very popular multicarrier transmission scheme. In fact, it has been adopted and/or proposed for several wideband applications such as Digital Video Broadcast (DVB) [1], Digital Audio Broadcast (DAB) [2], Local Area Wireless Networks (e.g. IEEE802.11, MMAC and HIPERLAN/2) [3] and Ultra Wide-Band (UWB) [4].

Due to its multicarrier nature, OFDM offers many advantages for wireless applications including good performance in multipath fading environments, high spectral efficiency and IFFT/FFT based transceiver implementations. However, one of the main disadvantages of multicarrier modulation is its high Peak-to-Average Power Ratio (PAPR) and consequently its vulnerability to non-linear distortion.

Currently, Radio-over-Fiber (RoF) is a technology that is being increasingly used to transport and deliver radio signals from remote antenna units (RAU) to a central station (CS) [5], [6], [7], including OFDM and Ultra-Wideband (UWB) signals [4]. In RoF systems, the electrical signal is converted into the optical domain using electro-optic (E/O) converters, such as Mach-Zehnder modulators (MZM) and laser diodes, and then sent through an optical fiber to an optical-electric (O/E) converter such as a PIN diode. RoF systems offer several potential benefits such as low loss, high bandwidth transmission and low complexity of the RAU, making these techniques suitable to transport and deliver high bitrate multi-carrier signals for long distances. However, these systems also suffer from specific impairments which include the nonlinearities associated with various optical components such as E/O converters. This type of impairment is particularly problematic for OFDM signals, so efficient techniques are on demand to counteract the nonlinear phenomena.

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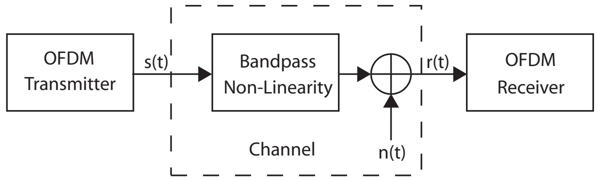

Fig. 1. Equivalent baseband OFDM communications system model.

In [8] and [9], we have derived the structure of the optimum receiver for non-linearly distorted OFDM signal. This optimum receiver consists of a set of correlators or a set of filters matched to every possible frequency component in the non-linearly distorted OFDM signal followed by a maximum likelihood (ML) detector by minimizing the error probability. In order to bypass complexity issues, we have also adopted a sub-optimal iterative procedure rather than the optimal exhaustive one associated with ML detection. We have previously demonstrated the potential of such optimum receiver in RoF uplink systems based on OFDM [9]. In this paper, in turn, we have derived the structure of minimum mean-squared error (MMSE) receiver for non-linearly distorted OFDM signal.

This paper is organized as follows: In section 2, we introduce the system model. Section 3 examines the structure of the MMSE receivers while section 4 examines its performance. Finally section 5 summarizes the main conclusions of the paper.

II. System Model

We consider the OFDM communications system model shown in Figure 1, where $s(t)$ is the complex envelope of the transmitted OFDM signal, $r(t)$ is the complex envelope of the received OFDM signal, $n(t)$ is the complex additive white Gaussian noise (AWGN) with power spectral density N_0, and the input-output characteristic $f(\cdot)$ of the bandpass

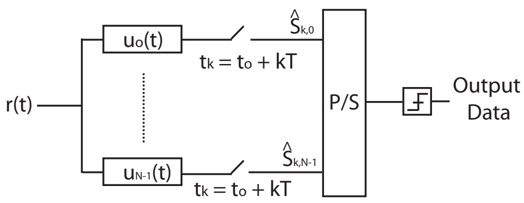

Fig. 2. Structure of the linear MMSE OFDM receiver.

non-linearity is represented by a "baseband" power series given by [10]

$$f(x) = \sum_{i=1}^{\infty} c_{2i-1} x |x|^{2i-2}. \tag{1}$$

We note in passing that this model can also be used to represent a typical RoF uplink system, where the bandpass non-linearity and additive white Gaussian noise represent the Mach-Zehnder Modulator (MZM) and the PIN photodiode thermal and shot noise, respectively.

The complex envelope of the transmitted OFDM signal is given by [8]:

$$s(t) = \sum_{k=-\infty}^{\infty} \sum_{n=0}^{N-1} S_{k,n} g_n(t-kT) \tag{2}$$

$$g_n(t) = \begin{cases} \frac{1}{\sqrt{T}} e^{j\frac{2\pi n t}{T}}, & \text{if } t \in [0,T] \\ 0, & \text{if } t \notin [0,T], \end{cases} \tag{3}$$

where $S_{k,n}$ is the complex transmitted symbol in time slot k and subcarrier n, N is the number of OFDM subcarriers and T is the OFDM symbols duration. This signal is distorted both by the nonlinearity and AWGN. So, the complex envelope of the received OFDM signal is given by:

$$r(t) = \sum_{i=1}^{\infty} c_{2i-1} s(t) |s(t)|^{2i-2} + n(t), \tag{4}$$

or

$$\begin{aligned} r(t) = & \sum_{i=1}^{\infty} c_{2i-1} \sum_{k=-\infty}^{\infty} \sum_{n_1=0}^{N-1} \cdots \sum_{n_{2i-1}=0}^{N-1} \\ & S_{k,n_1} \ldots S_{k,n_i} S^*_{k,n_{i+1}} \ldots S^*_{k,n_{2i-1}} \frac{1}{T^{i-1}} \\ & \times g_{n_1+\cdots+n_i-n_{i+1}-\cdots-n_{2i-1}}(t-kT) + n(t). \end{aligned} \tag{5}$$

The OFDM receiver will process the non-linearly distorted OFDM signal contaminated with noise in (4) or (5), in a linear fashion in order to optimize a performance metric, namely, the mean-squared error (MSE).

III. Structure of the MMSE Receiver

We now consider the structure of the linear MMSE receiver for non-linearly distorted OFDM signals in AWGN. We take the receiver to consist of a bank of N linear filters with impulse responses $u_n(t), n = 0, \cdots, N-1$, that produce estimates of the original information symbols (see Figure 2).

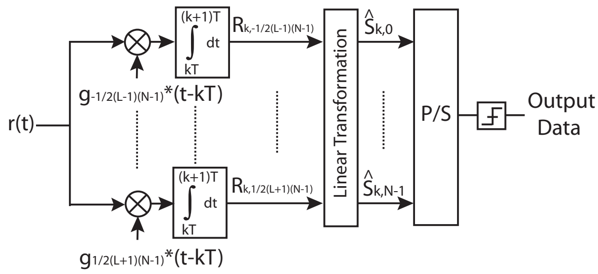

Fig. 3. Alternative structure of the linear MMSE OFDM receiver.

The objective is to determine the set of linear filters impulse response that minimize the linear MSE given by ξ

$$\begin{aligned} \xi &= E\left\{ \| \mathbf{S}_k - \hat{\mathbf{S}}_k \|^2 \right\} = \\ &= \sum_{n=0}^{N-1} E\left\{ \| S_{k,n} - \hat{S}_{k,n} \|^2 \right\} = \\ &= \sum_{n=0}^{N-1} E\left\{ \| S_{k,n} - [u_n(t_k) * r(t_k)] \|^2 \right\}, \end{aligned} \tag{6}$$

where $\mathbf{S}_k = [S_{k,n}; n = 0, \cdots, N-1]$ is the vector of information symbols in time slot k, $\hat{\mathbf{S}}_k = [\hat{S}_{k,n}; n = 0, \cdots, N-1]$ is the vector of estimates of information symbol in time slot k and "$*$" denotes the convolution. Representing the complex envelope of the received OFDM signal, $r(t)$, by its non-linear distorted and white Gaussian noisy components $y(t)$ and $n(t)$, respectively, (6) can be rewritten as

$$\begin{aligned} \xi &= \sum_{n=0}^{N-1} \Bigg[E\{S_{k,n} S^*_{k,n}\} + \\ &+ \iint u_n(\tau_1) u_n^*(\tau_2) E\{r(t_k-\tau_1) r^*(t_k-\tau_2)\} d\tau_1 d\tau_2 - \\ &- \int u_n^*(\tau) E\{S_{k,n} r^*(t_k-\tau)\} d\tau - \\ &- \int u_n(\tau) E\{S^*_{k,n} r(t_k-\tau)\} d\tau \Bigg] = \\ &= \sum_{n=0}^{N-1} \Bigg[E\{|S_{k,n}|^2\} + 2N_0 \int u_n(\tau) u_n^*(\tau) d\tau + \\ &+ \iint u_n(\tau_1) u_n^*(\tau_2) E\{y(t_k-\tau_1) y^*(t_k-\tau_2)\} d\tau_1 d\tau_2 + \\ &- \int u_n^*(\tau) E\{S_{k,n} y^*(t_k-\tau)\} d\tau - \\ &- \int u_n(\tau) E\{S^*_{k,n} y(t_k-\tau)\} d\tau \Bigg]. \end{aligned} \tag{7}$$

Using calculus of variations [11] it is possible to find the set of complex functions $u_n(t)$ that minimize the mean squared error, $\xi(u_n(t))$. From the Fréchet differential of ξ,

$$\frac{d}{d\alpha} \left\{ \xi\left(u_n(t) + \alpha h(t)\right)|_{\alpha=0} \right\} = 0, \quad \forall h(t), \tag{8}$$

it follows that the optimal linear filter responses satisfy the set of integral equations:

$$2\int u_n(\tau')E\{y^*(t_k-\tau)y(t_k-\tau')\}d\tau' + 2N_0u_n(\tau) = \\ = E\{S_{k,n}y^*(t_k-\tau)\}, n=0,\cdots,N-1. \quad (9)$$

Let us assume now that the non-linearity exhibits non-linear behavior of maximum order L (L is odd), i.e., in the model in (4) $c_{2i-1}=0$ for $i>(L+1)/2$. Consequently, it follows (from the integral equations) that the optimal linear filter responses are given by:

$$u_n(t) = \sum_{i=i_{min}}^{i_{max}} \alpha_{i,n} g_i^*(t_0-t), \quad n=0,\cdots,N-1, \quad (10)$$

where $i_{min}=-(L-1)(N-1)/2$, $i_{max}=(L+1)(N-1)/2$, and the coefficients $\alpha_{i,n}$, $i=-(L-1)(N-1)/2,\cdots,(L+1)(N-1)/2$, $n=0,\cdots,n-1$ correspond to the solution of a set of linear equations

$$\sum_{i=i_{min}}^{i_{max}} \alpha_{i,n}E\{Z_iZ_m^*\} + 2N_0\alpha_{m,n} = E\{S_{k,n}Z_m^*\}, \\ m=-\frac{1}{2}(L-1)(N-1),\cdots,\frac{1}{2}(L+1)(N-1), \quad (11)$$

where Z_m is a random variable given by:

$$Z_m = \int_{kT}^{(k+1)T} y(t)g_m^*(t-kT)dt = \\ = \sum_{i=1}^{\infty} c_{2i-1} \sum_{n_1=0}^{N-1}\cdots\sum_{n_{2i-1}=0}^{N-1} \\ S_{k,n_1}\cdots S_{k,n_i}S_{k,n_{i+1}}^*\cdots S_{k,n_{2i-1}}^*\frac{1}{T^{i-1}} \times \\ \times\ \delta(m-n_1-\cdots-n_i+n_{i+1}+\cdots+n_{2i-1}). \quad (12)$$

The expectations $E\{Z_iZ_m^*\}$ and $E\{S_{k,n}Z_m^*\}$ are the autocorrelation matrix of $\mathbf{Z} = [Z_{-(L-1)(N-1)/2},\cdots,Z_{(L+1)(N-1)/2}]$ and the cross-correlation matrix of $\mathbf{Z}$ and $\mathbf{S}$, respectively, given by

$$E\{Z_iZ_m^*\} = \\ = \sum_{l=1}^{\infty}\sum_{l'=1}^{\infty} c_{2l-1}c_{2l'-1}^* \sum_{n_1=0}^{N-1}\cdots\sum_{n_{2l-1}=0}^{N-1}\sum_{n'_1=0}^{N-1}\cdots\sum_{n'_{2l'-1}=0}^{N-1} \\ E\Big\{S_{k,n_1}\cdots S_{k,n_l}S_{k,n_{l+1}}^*\cdots S_{k,n_{2l-1}}^*S_{k,n'_1}^*\cdots \\ \cdots S_{k,n'_{l'}}^*S_{k,n'_{l'+1}}\cdots S_{k,n'_{2l'-1}}\Big\}\frac{1}{T^{l+l'-2}} \times \\ \times\delta(i-n_1-\cdots-n_l+n_{l+1}+\cdots+n_{2l-1}) \times \\ \times\delta(m-n'_1-\cdots-n'_{l'}+n'_{l'+1}+\cdots+n'_{2l'-1}) \quad (13)$$

and

$$E\{S_{k,n}Z_m^*\} = \sum_{l'=1}^{\infty} c_{2l'-1}^* \sum_{n_1=0}^{N-1}\cdots\sum_{n_{2l'-1}=0}^{N-1} \\ E\Big\{S_{k,n}S_{k,n_1}^*\cdots S_{k,n'_l}^*S_{k,n_{l'+1}}\cdots S_{k,n_{2l'-1}}\Big\}\frac{1}{T^{l'-1}} \times \\ \times\delta(m-n_1-\cdots-n'_l+n_{l'+1}+\cdots+n_{2l'-1}). \quad (14)$$

In order to obtain these correlations, we need to calculate the expected values $E\{\cdot\}$ in (13) and (14). Assuming that the complex transmitted symbols in each OFDM subcarrier are independent then [12]

$$E\Big\{S_{k,n_1}\cdots S_{k,n_l}S_{k,n_{l+1}}^*\cdots S_{k,n_{2l-1}}^*S_{k,n'_1}^*\cdots \\ \cdots S_{k,n'_{l'}}^*S_{k,n'_{l'+1}}\cdots S_{k,n'_{2l'-1}}\Big\} = \prod_{n'=0}^{N-1} E\left\{(S_{k,n'})^r(S_{k,n'}^*)^s\right\} \quad (15)$$

where r and s denote the number of elements in the form $S_{k,n'}$ and $S_{k,n'}^*$, respectively. Note that in (14) $l=1$.

As it can be seen, the derivation of the impulse responses of the linear filters $u_n(t)$ can be extremely complex in the computational sense, depending on the number of OFDM subcarriers, N, and the nonlinear order of the channel, L.

In a BPSK modulation scheme, complex transmitted symbols can take two possible values, $\{Ae^{j\pi b}; b=0,1\}$. Thus, assuming that complex transmitted symbols have equal probability, the expectations in (15) are given by

$$E\left\{(S_{k,n'})^r(S_{k,n'}^*)^s\right\} = \begin{cases} A^{(r+s)}, & r-s=0,\pm2,\pm4,\cdots \\ 0, & else. \end{cases} \quad (16)$$

Interestingly, we can appreciate the architecture of the linear MMSE receiver for non-linearly distorted OFDM signal in AWGN from two view points. On the one hand, we can see the MMSE receiver as consisting of a set of linear filters given by (10) and (11) (see also Figure 2). On the other hand, we can see the linear MMSE receiver as consisting of two stages (see Figure 3): the first stage consists of a set of correlators or a set of filters matched to every possible frequency component in the non-linearly distorted OFDM signal. This stage computes a set of sufficient statistics. The second stage is a simple linear transformation which is solely defined by the coefficients in (11). In this sense, the architecture of the linear MMSE receiver is identical to the architectures of the maximum likelihood receiver [8], [9], where the ML detector is substituted by the linear transformation and the decision devices.

IV. Results

This section examines the performance of the MMSE receiver for nonlinearly distorted OFDM signals in AWGN. In particular, we compare the performance of the linear MMSE receiver to that of the conventional receiver composed by a

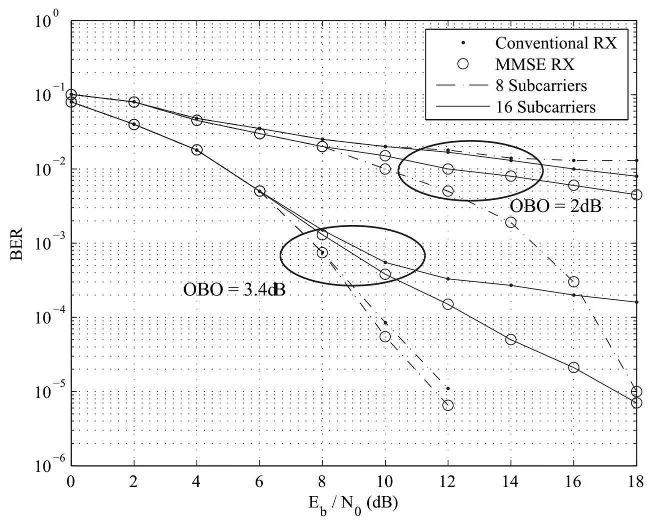

Fig. 4. Bit error rate (BER) as a function of E_b/N_0 for 8 and 16 subcarriers.

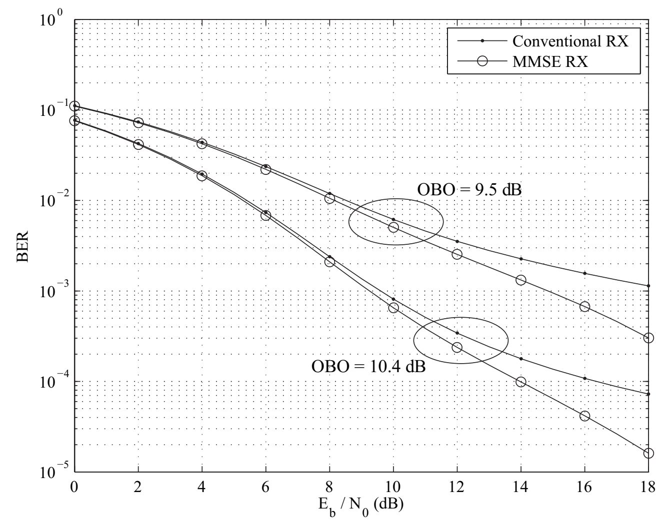

Fig. 5. Bit error rate (BER) as a function of E_b/N_0 for 48 subcarriers.

simple bank of N correlators matched to every OFDM subcarriers. We have conducted BER simulations for the system model in Figure 1. We have taken the non-linearity to be a simple third-order non-linearity where $c_1 = 1$, $c_3 = -0.75$ and $c_{2i-1} = 0$ for $i > 2$ because of the extremely high computational complexity associated with (13) and (14) when a higher nonlinear order is considered. We have also taken, for simplicity, the OFDM signal to be based on BPSK modulation, and with 8, 16 or 48 subcarriers. The BER performance is considered for different E_b/N_0 values and different output back-off (OBO). The OBO is given by

$$\text{OBO} = 10\log_{10}\frac{P_{OUT_{max}}}{\langle P_{OUT}\rangle}, \tag{17}$$

where

$$P_{OUT_{max}} = -\frac{1}{2}\left(\frac{16}{81}\right)\frac{c_1^3}{c_3}. \tag{18}$$

$P_{OUT_{max}}$ and $\langle P_{OUT}\rangle$ are the maximum power and the average power at the output of the non-linearity under consideration. This measure represents the degree of nonlinear strength.

Figures 4 and 5 show the bit error rate (BER) as a function of E_b/N_0 for 8, 16 and 48 OFDM subcarriers and different output back off (OBO) values. It can be seen that the MMSE receiver outperforms the conventional receiver. For small E_b/N_0 values there is no difference between the receivers, because the effect of the AWGN component is substantially higher than the nonlinear distortion effect. When the E_b/N_0 starts to increase, the AWGN component decreases and the nonlinear distortion effect becomes dominant. In this condition, MMSE receiver outperforms the conventional one considerably.

V. Conclusions

In this paper, we have derive the structure of the MMSE receiver for non-linearly distorted OFDM signals in AWGN.

This MMSE receiver exhibits a very simple structure consisting of a bank of correlators of filters matched to every possible frequency component in the non-linearly distorted OFDM signal followed by a linear transformation. Simulation results have shown that the MMSE receiver outperforms the conventional OFDM receiver.

With a current widespread interest in Radio over Fiber applications, where nonlinear phenomena associated with optical components is an important issue, we conjecture that this MMSE receiver is adequate to considerably improve performance, without incurring significant complexity penalties.

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