# Macro-Cellular Capacity Enhancement using Spatial Multiplexing MIMO with Linear Antenna Arrays

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*Abstract*—In this paper, we first introduce a Frequency Selective (FS) Multiple Input Multiple Output (MIMO) framework for macro-cells in a realistic urban environment. Next, MIMO configuration characteristics are investigated in order to maximize capacity, mainly the number of antennas and inter-antenna spacing. Channel and capacity simulation results are presented for the city of Lisbon, Portugal, using different antenna configurations. The results suggest optimized MIMO configurations, considering the antenna array size limitations, at the Mobile Station (MS) side.

*Index Terms*—MIMO, Channel Model, Antenna Arrays, Water-filling.

#### I. INTRODUCTION

The capacity advantage of MIMO [1],[2] channels lays in the decomposition of the channel into several spatial subchannels, each one with different gain. The offered capacity depends directly on the sub-channel power distribution, MIMO configuration (antenna array characteristics), and Signal-to-Noise-Ratio (SNR).

If the channel is unknown at the Transmitter (TX), uniform power allocation between the antenna elements is optimal in the capacity sense, and serious capacity gain is available compared with Single Input Single Output (SISO) systems. However, if the channel is known at the Receiver (RX) and also at the TX, the channel capacity can be further increased by allocating the power in a more efficient manner, the optimum power allocation following the Shannon's principle of waterfilling [3].

In this paper we introduce a new Frequency Selective (FS) MIMO framework for macro-cells in a realistic urban environment. In the following, the MIMO configuration characteristics are studied in order to maximize capacity, mainly the number of antennas and inter-antenna spacing.

A description of antenna system and mobility dependent MIMO channel model is given in section II, followed by the FS MIMO capacity review in section III. Section IV presents the model implementation over the Lisbon urban environment along with the MIMO configuration impact in section V. Finally, the conclusions are presented in section VI.

#### II. THE MIMO CHANNEL MODEL

The MIMO concept is defined as a radio link with R antennas at the Base Station (BS) and S antennas at the MS. The received signal vector  $\mathbf{y}(f)$  at the MS antenna array is denoted by,

$$\mathbf{y}(f) = [y_1(f), y_2(f), ..., y_S(f)]^T$$
(1)

where  $y_s(f)$  is the signal at the  $s^{th}$  antenna element,  $[...]^T$  denotes the transpose operation and f is the carrier frequency. Similarly, the transmitted signals at the BS,  $x_r(f)$ , define the vector  $\mathbf{x}(f)$ ,

$$\mathbf{x}(f) = [x_1(f), x_2(f), ..., x_R(f)]^T.$$
 (2)

The vectors  $\mathbf{y}(f)$  and  $\mathbf{x}(f)$  are linked by the following expression,

$$\mathbf{y}(f) = \mathbf{H}(f)\mathbf{x}(f) + \mathbf{n}(f)$$
(3)

where  $\mathbf{n}(f)$  is additive white Gaussian noise and  $\mathbf{H}(f) \in \mathbf{C}^{S \times R}$  is the instantaneous MIMO radio channel matrix, i.e., refers to a radio propagation channel snapshot.  $\mathbf{H}(f)$  describes the connections between the BS and the MS and depends directly on the Radio Environment (RE).

The RE characterization will be set by the extended European COoperation in the field of Scientific and Technical research (COST) 273 Directional Channel Model (DCM) [4]. According to the model, the received signal at the MS consists of several time-delayed multi-path replicas of the transmitted signal which are grouped in clusters. The first cluster is always around the MS. The N clusters are defined by powers, delays, Angle of Departure (AoD), Angle of Arrival (AoA) and are determined stochastically according to the channel model generation procedure. Each cluster consists of M Multi-Path Component (MPC)s.

The BS is assumed to be multi-sectored. Each sector is equipped with a multi element array antenna which, along with the MS array, will define the MIMO setup.

The MS travels along several BSs service area. In a certain time instant, a BS sector will be the serving one, according to a classic signal strength based locating algorithm.

In order to calculate the MIMO channel matrix  $\mathbf{H}$ , an evaluation technique that requires only a single measurement of the channel is proposed [5].

Based on this technique, the developed MIMO channel model is presented. For an R element linear BS array and a S element linear MS array, the channel coefficients for Nclusters, each one of them with M MPCs, are given by a  $S \times R$  matrix of complex amplitudes. The  $(s, r)^{th}$  component (s = 1, ..., S; r = 1, ..., R) of **H** is calculated generating different realizations of the transfer function from the  $r^{th}$  TX antenna to the  $s^{th}$  RX antenna as,

$$h_{s,r}(f) = \sum_{l=1}^{L} \sum_{n=1}^{N} \sum_{m=1}^{M} a_{n,m} \sqrt{G_{BS,r}\left(\theta_{n,m,AoD}\right)}$$
$$x \exp\left(-jkd_r \sin\left(\theta_{n,m,AoD}\right)\right) \sqrt{G_{MS,s}\left(\theta_{n,m,AoA}\right)}$$

×

$$\times \exp\left(-jkd_{s}\sin\left(\theta_{n,m,AoA}\right)\right)\exp\left(-j\,2\pi f\tau_{l}\right)$$
$$\times \exp\left(-jk\left\|\vec{\mathbf{v}}\right\|\cos\left(\theta_{n,m,AoA}-\theta_{v}\right)\right) \times g\left(\tau_{n,m},\tau_{l}\right)$$
(4)

where, L is the number of delay taps;  $\theta_{n,m,AoD}$  is the absolute AoD for the  $m^{th}$  (m = 1...M) MPC of the  $n^{th}$  cluster at the BS with respect to the BS broadside;  $\theta_{n,m,AoA}$  is the absolute AoA for the  $m^{th}$  (m = 1...M) MPC of the  $n^{th}$  cluster at the MS with respect to the MS broadside;  $G_{BS,r}(\theta_{n,m,AoD})$ is the  $r^{th}$  BS antenna gain pattern of each array element;  $G_{MS,s}(\theta_{n,m,AoA})$  is the  $s^{th}$  MS antenna gain pattern of each array element;  $a_{n,m}$  is the complex amplitude of the  $(n,m)^{th}$ MPC; j is the square root of -1; k is the wave number  $2\pi/\lambda$  where  $\lambda$  is the carrier wavelength in meters;  $d_r$  is the distance in meters from BS antenna element r to the reference antenna (for the reference antenna  $r = 1, d_1 = 0$ );  $d_s$  is the distance in meters from MS antenna element s to the reference antenna (for the reference antenna  $s = 1, d_1 = 0$ );  $\|\vec{\mathbf{v}}\|$  is the magnitude of the MS speed vector;  $\theta_v$  is the angle of the MS speed vector;  $g(\tau_{n,m},\tau_l)$  is an auxiliary function given by

$$g(\tau_{n,m},\tau_l) = \begin{cases} 1 & \tau_{n,m} \in [\tau_{l-1},\tau_l] \\ 0 & other \end{cases}$$
(5)

where  $\tau_{n,m}$  is the  $(n,m)^{th}$  MPC excess delay referred to the Line of Sight (LoS) radio path and  $\tau_l$  is the  $l^{th}$  tap delay which is entirely dependent on the system bandwidth, B, as

$$\tau_l = \frac{1}{B}.\tag{6}$$

The BS antenna gain pattern used for each antenna element, for both downlink and uplink, is extracted from [6].

The proposed MIMO channel model is therefore, system, radio environment and mobility dependent. The MIMO channel will depend on the system configuration: number of antennas, antenna spacing, antenna radiation pattern and system bandwidth. The radio environment realization is introduced using a realistic macro-cell directional model [4]. Additionally, the MS mobility [7] guarantees a realistic navigation through the BS service area. The Doppler shift is introduced using the speed vector information.

# III. THE FS MIMO CHANNEL CAPACITY

The concept of capacity in FS MIMO channels is investigated in this section. The presented values correspond to the theoretical maximum amount of information that can be transmitted over a bandwidth limited channel.

The channel bandwidth is split into Q flat-fading subchannels  $\mathbf{H}(f_q)$ , where  $f_q$  is the  $q^{th}$  sub-carrier central frequency. The sub-carrier bandwidth will be smaller than the coherence bandwidth.

The number of spatial sub-channels over a radio link can be computed by using Singular Value Decomposition (SVD) of matrix  $\mathbf{H}(f_q)$  or the Eigenvalue Decomposition (EVD) [8] of the instantaneous correlation matrix  $\mathbf{R}(f_q)$  defined as,

$$\mathbf{R}(f_q) = \begin{cases} \mathbf{H}(f_q) \mathbf{H}(f_q)^H, S \le R\\ \mathbf{H}(f_q)^H \mathbf{H}(f_q), R < S \end{cases}$$
(7)

where  $\left[ \ldots \right]^{H}$  represents conjugate transposition.

Irrespective of the numerical method used to perform the analysis, a channel matrix  $\mathbf{H}(f_q)$  may offer  $K_q$  parallel subchannels with different power gains,  $\alpha_{k,q}$ , where

$$K_q = Rank(\mathbf{R}(f_q)) \le Min(S, R) \tag{8}$$

and the functions Rank() and Min() return the rank of the matrix and the minimum value of the arguments.

In the situation where the channel is known at both TX and RX and is used to compute the optimum weight, the power gain in the  $(k, q)^{th}$  eigenmode channel is given by  $\alpha_{k,q}$  eigenvalue. In order to evaluate the performance of different power allocation methods, we use the normalized Shannon channel capacity (in bit/s/Hz). For the  $k^{th}$  eigenmode on the  $q^{th}$  sub-channel, it can be expressed as [3],

$$C_{k,q} = \sum_{k=1}^{K_q} \log_2\left(1 + \alpha_{k,q} \frac{p_{k,q}}{\sigma_N^2}\right) \tag{9}$$

where  $p_{k,q}$  is the power assigned to the  $(k,q)^{th}$  sub-channel, and  $\sigma_N^2$  is the noise power.

Hence, the total normalized MIMO channel capacity for a FS sub-channel is,

$$C = \frac{1}{Q} \sum_{q=1}^{Q} \sum_{k=1}^{K_q} \log_2\left(1 + \alpha_{k,q} \frac{p_{k,q}}{\sigma_N^2}\right).$$
 (10)

Given the set of eigenvalues  $\alpha_{1,q}$ ,  $\alpha_{2,q}$ , ..., $\alpha_{K_q,q}$  the power  $p_{k,q}$  allocated to each spatial sub-channel (k,q) can be determined to maximize the capacity by using Gallager's waterfilling theorem. This will be named as spatial water-filling. The algorithm assigns equal power to all the Q sub-channels considering the power constraint,

$$\frac{P_{Tx}}{Q} = \sum_{k=1}^{K_q} p_{k,q}$$
 (11)

where  $P_{Tx}$  is the total transmitted power.

The water-filling is of course dependent on the knowledge of the channels on the transmitter side. In the case where the channel is unknown at the TX, the only reasonable division of power is a uniform distribution over the antennas and frequencies, i.e.,

$$p_{k,q} = \frac{P_{Tx}}{R \cdot Q} \quad . \tag{12}$$

# IV. MIMO CHANNEL MODEL IMPLEMENTATION

The MIMO channel model simulator was developed in Matlab<sup>©</sup> and requires building data information, terrain data, street data, BS and MS information. The BS and MS data store the geographical positioning, height, power parameters, MIMO antenna configuration, radiation pattern, and system parameters like handover hysteresis and offset. The MS positional data is generated using a new mobility model developed and validated [7] for Lisbon city.

The considered DCM [4], used as an input to the MIMO frequency selective channel construction, imports cartographic data to dynamically introduce terrain and clutter effects on path-loss prediction [9] and MPC cluster generation [10]. For each BS-MS radio-link, the DCM is applied. The MS local



Fig. 1. MIMO and water-filling gain for MIMO  $m \times m$  setup, m = 1, ..., 7. SNR = 5 dB, half of wavelength antenna spacing.

movement is compared with the size of the local area, which is defined as the area over which all large-scale fading parameters can be viewed as constants. The large-scale parameters are updated when the MS movement exceeds the size of the local area, which was set to  $20\lambda$ , being  $\lambda$  the carrier wavelength.

The estimated minimum coherence time is 4.8 ms, which gives a 3 cm channel coherent travel distance at 20 km/h average speed. The speed was calculated from the mobility model. In the following, for each  $20\lambda$  local area size (at 2 GHz), 100 channel realizations were generated.

The eigenvalue and capacity calculations were performed for Q = 100 sub-channels with a 200 KHz individual bandwidth. Hence, the total bandwidth *B* is 20 MHz. The sub-channel bandwidth is equal to the channel coherence bandwidth considering a 1  $\mu$ s calculated delay spread.

The tested urban RE is the city of Lisbon, Portugal, and several virtual test MSs travel around the city for a period of 10 minutes. To this end, a very large number of MIMO channel realizations were generated for each cluster oriented large-scale fading set of parameters.

#### V. MIMO CONFIGURATION IMPACT

The current section presents the the MIMO configuration impact on channel capacity. The considered MIMO parameters are the number of antennas and BS/MS antenna spacing.

## A. Number of Antenna Elements Impact

Several half wavelength  $(0.5\lambda)$  spaced MIMO configurations were chosen and compared with the reference SISO. The BS and MS number of antenna elements is equal and varies from one to seven. The MIMO gain is defined as the relation between the MIMO configuration mean capacity and the SISO mean capacity. Fig. 1 presents the MIMO gain using the two implemented power allocation techniques. The waterfilling gain relates, for each MIMO configuration, the mean capacity using spatial water-filling with the equivalent mean capacity using an uniform power allocation.

The MIMO gain increases with the number of antenna elements. Starting with an already outstanding 80% increase



Fig. 2. MIMO and water-filling gain for MIMO  $3(m\lambda) \times 3(0.5\lambda)$  setup, SNR = 5 dB, BS antenna spacing variation.

using a simple  $2 \times 2$  MIMO configuration, the mean capacity gain can reach 380% using MIMO  $7 \times 7$  with spatial waterfilling.

The capacity gain from spatial water-filling compared to the capacity from uniform power allocation is considerable (a 30 % increase was noticed), for all the configurations.

The waste of power when using uniform power allocation is more severe when really poor eigen-mode channels are evident, like the presented macro-cell case, where the second eigen-channel can be 20 dB smaller than the first one. In fact, the capacity gain from water-filling increases with the difference between the eigenvalues.

#### B. BS Antenna Spacing

For the BS antenna element spacing study, the MS antenna spacing is kept at half of the wavelength  $(0.5\lambda)$ . The 3rd Generation Partnership Project (3GPP) [6] suggests several BS antenna spacings from  $0.5\lambda$  to  $10\lambda$ . Hence, the proposed test is to vary the BS antenna spacing from  $0.25\lambda$  to near  $10\lambda$  analyzing several MIMO configurations.

In terms of antenna elements number, the MIMO  $3 \times 3$ setup is presented. Other configurations were tested and the conclusions are similar. The eigen-analysis for the BS antenna spacing changes revealed that the maximum eigenvalues  $\alpha_{1,q}$ do not suffer any major variation with the element spacing increase. In the remaining eigenvalues ( $\alpha_{2,q}, \alpha_{3,q}$ ), it exists an increase with element separation, specially from 0.5 $\lambda$  to  $5\lambda$  BS antenna spacing configurations. For antenna separations higher that  $5\lambda$ , the secondary eigenvalues still increase with antenna spacing, but at a lower rate.

The results are displayed in Fig. 2. The MIMO gain with uniform power allocation shows an increase with the BS antenna spacing. The MIMO gain then saturates for BS antenna spacings higher than  $5\lambda$  for both MIMO configurations.

In the water-filling case, the MIMO gain is approximately constant. The power transference to the strongest and constant eigenmodes  $(\alpha_{1,q})$ , removes the moderate capacity increase effect due to the secondary eigenvalues growth with the BS antenna spacing. As a final remark, the results suggest that, for the macro-cell case, if no channel information is available, BS antenna spacing should not exceed  $5\lambda$ , since no additional gain increase was detected using uniform power allocation. Additionally, if channel information is provided and spatial water-filling is used, the BS antenna spacing can be a lot lesser, with no apparent degradation on offered channel capacity.

#### C. MS Antenna Spacing

For the MS antenna element spacing impact, the BS antenna spacing is kept at  $5\lambda$ . The 3GPP [6] suggests a typical MS antenna spacing of  $0.5\lambda$ . The intention is to get capacity results with similar antenna spacings in a macro-cell urban environment, in order to check if this value is adequate. Having in mind the MS size limitations, the antenna separation should not exceed, say, one wavelength,  $\lambda$ .

The maximum separation depends naturally on the number of antenna elements, in order to keep the MS antenna array under a realistic size.

The following results are for a MIMO  $2 \times 2$  setup. The eigen-analysis for the MS antenna spacing changes shows that the maximum eigenvalues  $\alpha_{1,q}$  do not suffer any major variation with the element spacing increase. In the secondary eigenvalues  $(\alpha_{2,q})$ , it exists an increase with element separation, specially from  $0.1\lambda$  to  $0.3\lambda$  MS antenna spacing configurations. For MS antenna separations higher that  $0.3\lambda$ , the secondary eigenvalues still increases with antenna spacing, but much more slowly. As in the BS antenna spacing analysis, the MS antenna spacing mean MIMO and water-filling gains are presented in Fig. 3.

Looking at the values for the MIMO gain with uniform power allocation, the data shows an increase with the MS antenna spacing. The MIMO gain stabilizes for MS antenna spacings higher than  $0.4\lambda$ .

In the water-filling case, the MIMO gain also saturates for higher antenna spacings. For shorter antenna spacings (shorter than  $0.3\lambda$ ), the  $(\alpha_{2,q})$  eigenvalue is very weak, which enables an increased water-filling gain compared with uniform power distribution, since practically no waste of power is done on the secondary eigenvalues.

The capacity MIMO gains reach a 60 and 70% increase for the uniform and water-filling power allocation, respectively. The water-filling gain is aproximately 8%.

The simulated data agrees with the 3GPP suggested  $0.5\lambda$  value [6]. With or without smart power allocation techniques and for these SNR conditions, there is no additional advantage on using MS antenna spacings larger than half of the wavelength,  $0.5\lambda$ .

Considering the MS terminal size limitations, the simulations show that is preferable to maintain small element spacings of  $0.5\lambda$  and if possible add more elements, than maintain the number of elements and increase antenna spacing. The intense angular muti-path dispersion in the MS surroundings enables the  $0.5\lambda$  value or even slightly lesser antenna spacing. On the other hand, smaller antenna spacings (less than  $0.3\lambda$ ) should not be implemented at the expense of considerable capacity reduction.



Fig. 3. MIMO and water-filling gain for MIMO  $2(5\lambda) \times 2(m\lambda)$  setup, SNR = 5 dB, MS antenna spacing variation.

## VI. CONCLUSIONS

In this paper a new FS MIMO framework for macro-cells in a realistic urban environment is introduced. The MIMO model is built over a previously developed channel model, which is an extension of COST 273 channel model, introducing terrain and building information into radio channel simulation.

Next, MIMO configuration characteristics are investigated in order to maximize capacity, mainly antenna element number and inter-element spacing. Channel and capacity simulation results are presented for the city of Lisbon, Portugal, using different antenna configurations. Two power allocations schemes are considered, uniform distribution and FS spatial waterfilling . The results suggest optimized MIMO configurations, considering the antenna array size limitations, mostly on the MS side.

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