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EXTRACTION OF LASER RATE EQUATIONS PARAMETERS

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ABSTRACT

A technique for the extraction of laser rate equation parameters to be used in the simulation of high-speed optical telecommunication systems is described. The simulation using the extracted parameters exhibits good agreement with the experimental results, even for a large current modulation and soliton pulses. A model, which represents the parasite effect introduced by the mount fixture, was built. The model presented here is ready to use in the simulation of high-speed lightwave systems.

Keywords: Optical Communications, Optoelectronics Characterisation, Simulation

1. INTRODUCTION

For the purpose of optical telecommunication systems simulation, models based in a rate equation description of semiconductor lasers, representing the time variation of the optical power and the phase of the electrical field at the laser output, as function of the injection current, are available. To use these models, values for the rate equations parameters must be chosen appropriately in order to obtain agreement between simulated and measured results for system performance.

In these paper we present a technique to obtain the rate equation parameters for a set of simple laboratory measurements. This set of measurements is based on the small signal intensity modulation (IM) frequency response of the semiconductor laser. The small signal model characterises the laser modulation dynamics. However, accurate values for the resonance frequency and damping factor are difficult to obtain from the direct measurement of the laser frequency response, due to the parasite effects introduced by the mount fixture. In this contribution we also present a technique based in the ratio of two laser transfer functions, for different bias current, which eliminates the parasite constrains and allows an accurate laser rate equation parameters estimation.

During this work and for the entire laboratory measurements we used a commercial semiconductor distributed feedback (DFB) laser, with a central wavelength of 1550 nm.

2. LASER MODEL

The laser dynamics can be modelled by coupled rate equations [1] which describe the relation between the carrier number $N_p(t)$, the photon density $S_p(t)$ and the optical phase $f(t)$.

$$\frac{dN_p(t)}{dt} = \frac{I(t)}{q} - \frac{N_p(t)}{\tau_n} - g_{po}\frac{N_p(t) - N_{pt}}{1+\varepsilon_p S_p(t)} S_p(t) \tag{1}$$

$$\frac{dS_p(t)}{dt} = g_{po}\frac{N_p(t) - N_{pt}}{1+\varepsilon_p S_p(t)} S_p(t) - \frac{S_p(t)}{\tau_p} + \beta_s \frac{N_p(t)}{\tau_n} \tag{2}$$

$$\frac{d\Phi(t)}{dt} = \frac{\alpha_H}{2} g_{po}\left[N_p(t) - N_{pt}\right] \tag{3}$$

N_{pt} is the carrier number at transparency, t_p is the photon lifetime, t_n is the carrier lifetime, b_s is the spontaneous emission factor, e_p is the gain compression factor, g_{po} is the gain slope constant, $I(t)$ is the injected current, a_H is the linewidth enhancement factor and q is the electron charge. The output power and the threshold current are given by:

$$P(t) = \frac{\eta h \upsilon}{\tau_p} S_p(t) \tag{4}$$

$$I_{th} = \frac{q}{\tau_n}\left(N_{pt} + \frac{1}{g_{po}\tau_p}\right) \tag{5}$$

Where h is the quantum efficiency, h the Planck constant and n the radiation frequency. The small signal IM frequency response of a semiconductor laser determined from the rate equations is given by equation 6. Subtracting the frequency response for a bias current just above threshold, $H(f{:}Y_0{:}Z_0)$, from the frequency response for a bias current well above threshold, H(f:Y:Z), the parasite effects introduced by the mount fixture are eliminated [2], see equation 7.

$$H(f:Y:Z) = \frac{Z}{\left(Z - (2\cdot\pi\cdot f)^2 + j\cdot 2\cdot\pi\cdot f\cdot Y\right)} \tag{6}$$

$$S(f) = 20\log\left|\frac{H(f:Y:Z)}{H(f:Y_0:Z_0)}\right| \tag{7}$$

Where Y and Z are functions of the laser parameters and of the laser bias current are given by:

$$Y = g_{po}\frac{S_p}{1+\varepsilon_p S_p} + \frac{1}{\tau_n} - g_{p0}\frac{(N_p - N_{pt})}{(1+\varepsilon_p S_p)^2} + \frac{1}{\tau_p} \tag{8}$$

$$Z = g_{p0}\frac{S_p}{1+\varepsilon_p S_p}\frac{1}{\tau_p} + (\beta_s - 1)\frac{g_{p0}}{\tau_n}\frac{(N_p - N_{pt})}{(1+\varepsilon_p S_p)^2} + \frac{1}{\tau_p\tau_n} \tag{9}$$

3. PARAMETER EXTRACTION

The frequency responses for different bias currents where measured using a network analyser, between 0.04 and 12.50 GHz. The laser threshold current is 11.55 mA. Figure 1 shows the subtraction of two frequency responses and the theoretical fit to equation 7, using a Marquardt-Levenberg algorithm, giving the results of figure 2. The sum of squared errors between the values of Y, Z, P, I_{th} in equations 4, 5, 8, 9 and the measured values from figure 2 are minimised for each bias current. The minimisation is simultaneous for the four equations and over the 9 parameters from the rate equations. For the minimisation, a quasi-Newton method and a finite-difference gradient was used, the initial estimates were chosen from a set of physical reasonable values adequate to a DFB laser.

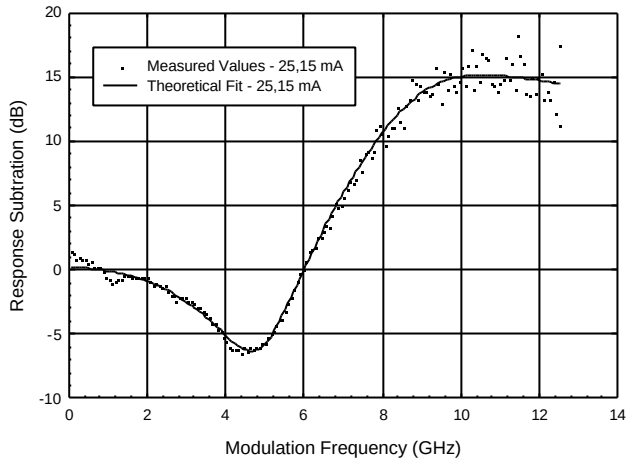


Figure 1. Response Frequency Subtraction

	Bias Current (mA)						
Parameter	15.00	20.00	25.00	30.00	35.00	40.00	45.00
P(mW)	0.408	0.860	1.293	1.745	2.166	2.580	2.995
Y ($x10^{10}$ s^{-1})	-	2.192	3.149	4.544	5.283	6.273	6.713
Z ($x10^{21}$ s^{-2})	-	2.255	3.548	4.918	5.872	7.463	7.578
Y_0 ($x10^{10}$ s^{-1})	1.235	-	-	-	-	-	-
Z_0 ($x10^{21}$ s^{-2})	0.941	-	-	-	-	-	-

Figure 2. Measured Parameters for different bias current.

Since the values from the minimisation have a small dependence on the bias current (except N and S which depend on the current), a second minimisation was made using these values as initial estimates. The final values have very little dependence on the bias current, which can be attributed to experimental uncertainty. The values for the constant parameters of the rate equations are shown in figure 3.

g_{po} ($x10^4 s^{-1}$)	t_p ($x10^{-12}$ s)	t_n ($x10^{-10}$ s)	b_s ($x10^{-6}$)	e_n ($x10^{-7}$)	N_{pt} ($x10^6$)	h
4.161	3.294	2.094	7.611	1.712	7.102	0.121

Figure 3. Extracted parameters

The a_H parameter, which was not determined by this method, could be estimated from the measurement of the joined frequency response of a direct modulated laser and a dispersive medium[3]. In our case yielded to a value of 2.981.

4. SIMULATION

To verify the robustness of these values, the theoretical solution of the rate equations was compared with experimental results. A model, which represents the rate equations, was built in a mathematics program named *Matlab*. The simulated and real values from the optical power as function of the bias current are shown in figure 4.

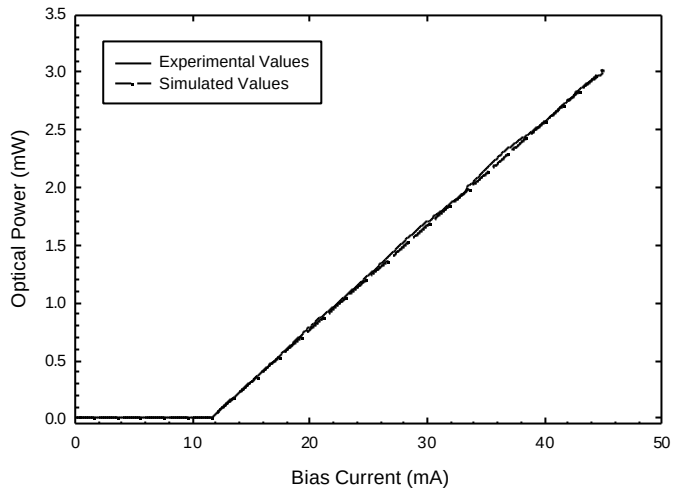


Figure 4. DC Response

The simulated and measured optical power for a laser working with a fixed bias current and a RF modulation signal are shown in figure 5. A reference from the RF modulation signal was captured with *Labview* and used as the modulation current in the simulation. In figure 6 the values for the optical frequency shift in a modulated laser are shown. A Mach-Zehnder interferometer has been used as an optical discriminator to measure the optical frequency shift [4].

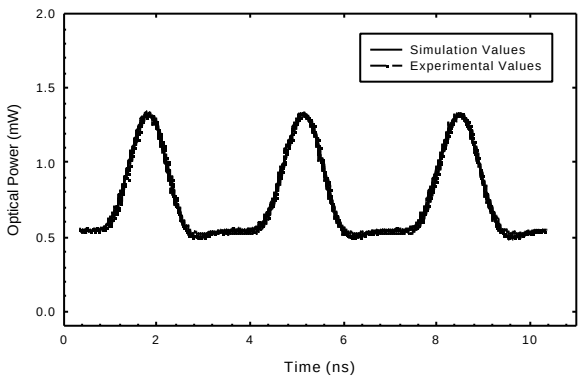


Figure 5. AC Response

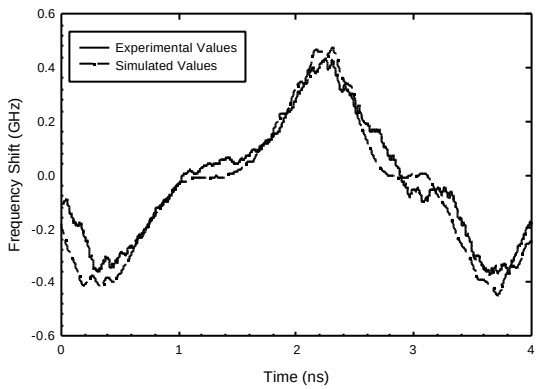
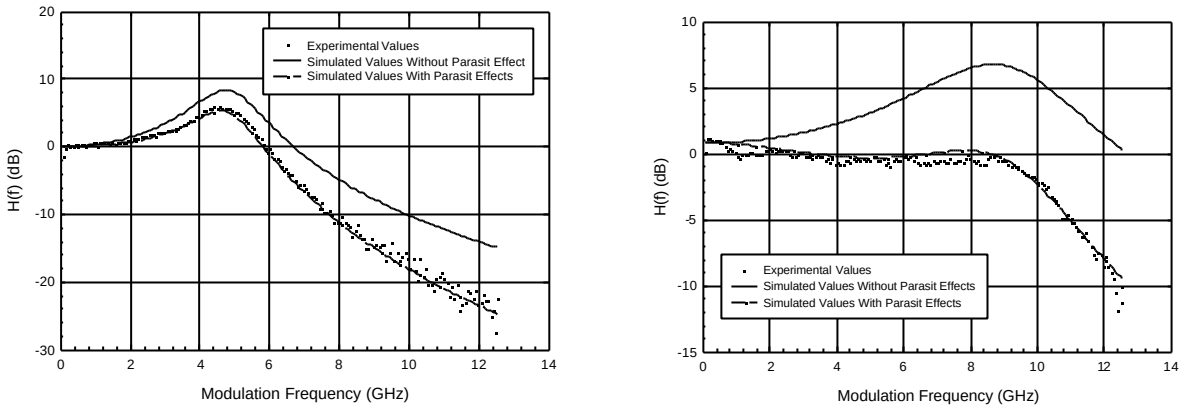


Figure 6. Optical Frequency Shift

5. MODEL FOR THE PARASITE EFFECTS

The frequency response measured directly in lab from the laser, is the joined frequency response of the semiconductor laser, given by the equation 6, with the frequency response of the mount fixture. Assuming that the mount fixture frequency response is independent of the bias current, and the parasite effects are mainly capacitive and resistive, the parasite frequency response can be approximated by a first order filter. After adjusting the parameters of a RC filter we obtained a -3 dB bandwidth of 4.8 GHz for the parasite effects. In figure 7 we present the experimental values of the laser frequency response obtained directly in the lab, and the simulated values with and without the RC filter.



(a) (b)

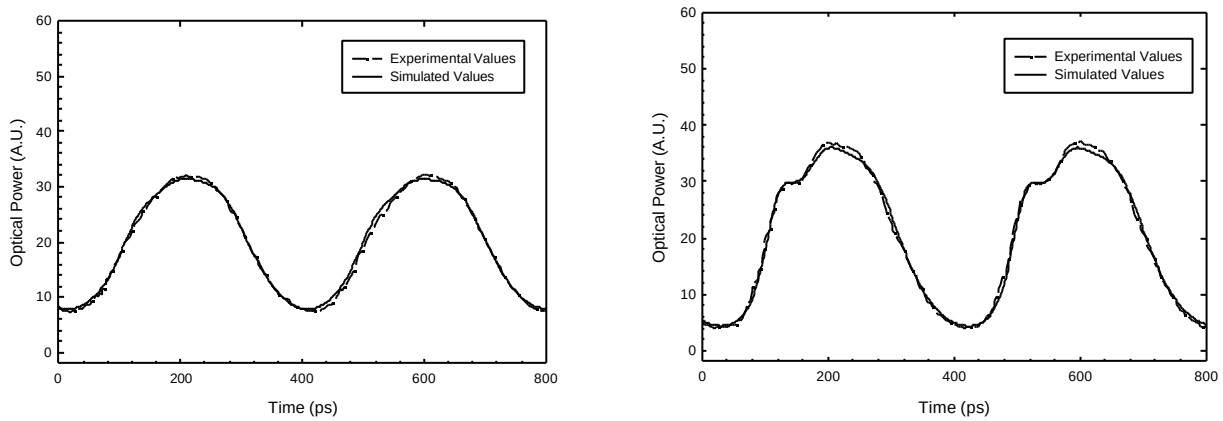
Figure 7. Experimental and simulated values from the laser frequency response, for different bias currents a) 15 mA and b) 25 mA.

6. LARGE SIGNAL MODULATION

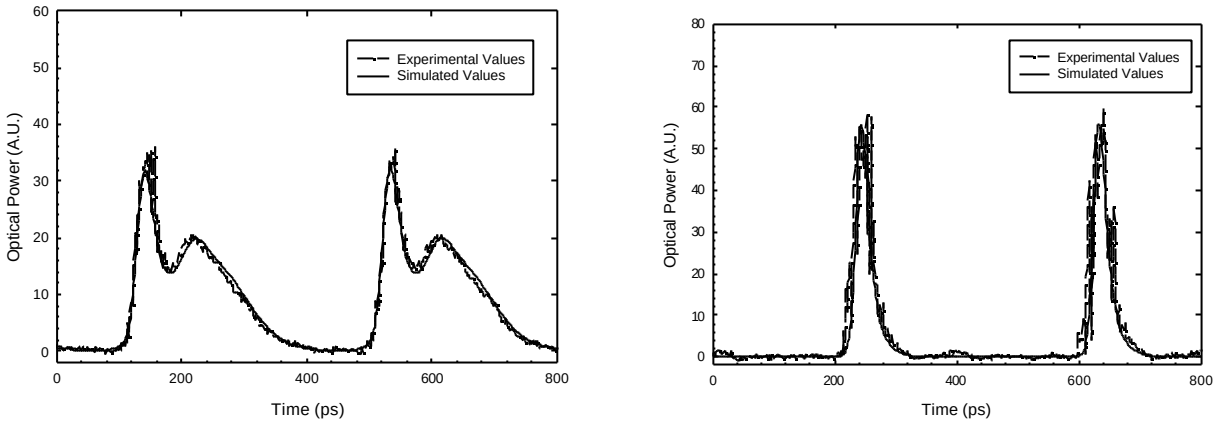
By operating the laser on a non-linear regime of gain switching, it is possible to obtain very short optical pulses[5], suitable for high-speed lightwave telecommunication systems, from a quasi-sinusoidal modulation current. In these operation regime the small signal laser model, presented in section 2, cannot be used and we have to solve numerically the rate equations to obtain the optical power and the phase of the electrical field at the laser output.

Defining the modulation index as the ratio of the modulation current and the difference between the bias and the threshold current, we can obtain different pulses width just by adjusting the bias current. In figure 8 we can see the experimental and simulated pulses for different modulation indexes.

We can see good agreement between the simulated results and the measurements obtained in the lab. The shorter pulses obtained in figure 8-d, are to be used in a soliton telecommunication system working at 10 Gbit/s.



(a) (b)



(c) (d)

Figure 8. Experimental and simulated optical pulses for different modulation indexes.
a) m = 0.87, b) m = 1.1, c) m = 1.5 and d) m = 6.6

7. CONCLUSION

We presented a technique to obtain the rate equation parameters through the measurement of the threshold current, the optical power and the frequency response of a semiconductor laser.

We used a model for the parasite effects, introduced by the mount fixture, based on a RC filter that presents a good agreement with experimental measurements.

This laser numerical model was used to simulate the laser dynamics with large current modulation, and in a gain switching operation mode. A good agreement was obtained between the simulated and the experimental values even for soliton pulses.

8. ACKNOWLEDGEMENT

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