

# Noise-Induced Spectral Shifts in Pseudo-Linear Fiber-Optic Communication Systems

Armando N. Pinto and Govind P. Agrawal

*Dept. Elect., Telecom., and Informatics, University of Aveiro, and Institute of Telecommunications, 3810-193 Aveiro, Portugal*  
*Institute of Optics, University of Rochester, Rochester, New York, 14627*  
*anp@det.ua.pt, gpa@optics.rochester.edu*

**Abstract:** Signal and noise interaction mediated by the Kerr effect in fibers produces random pulse central frequency shifts. We show that strong correlation between the signal and noise evolution makes this effect noteworthy even in pseudo-linear systems.

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The interaction between the signal and noise, mediated by the Kerr effect, was firstly studied in the context of solitons [1]. In this case, pulses behave like particles. The use of perturbation theory shows that noise-induced random shifts in the soliton central frequency lead to timing jitter that grows cubically with the distance. A cubic growth of timing jitter was also found in the case of linear pulses [2], even though signal and noise do not interact along the transmission channel in this case. As far as we know, the interaction between signal and noise, mediated by the Kerr effect, has not yet been studied in the case of pseudo-linear pulses, even though this regime is a strong candidate for the next generation of transmission systems operating at 40 and 160 Gbit/s. As pseudo-linear pulses do not retain the particle-like nature of solitons, a different method must be used for them. Here, we present a general method suitable for linear, pseudo-linear, and nonlinear pulses. Our method also permits visualization of the noise-induced central frequency shifts all along the transmission line.

Signal propagation in optical fibers can be modelled by the nonlinear Schrödinger equation (NLSE),  $\partial A/\partial z + i(\beta_2/2)\partial^2 A/\partial t^2 + \alpha A/2 = i\gamma|A|^2 A$ , where  $A$  is normalized such that  $|A|^2$  gives the optical power,  $\beta_2$  accounts for chromatic dispersion,  $\alpha$  for optical attenuation, and  $\gamma$  for the Kerr effect [3]. As we intend to study the interaction between signal and noise, we replace  $A$  in the NLSE by  $S + N$ , and find that the signal  $S$  and noise  $N$  satisfy the following two equations:

$$\frac{\partial S}{\partial z} + i\frac{\beta_2}{2}\frac{\partial^2 S}{\partial t^2} + \frac{\alpha}{2}S = i\gamma|S|^2 S + i\gamma SN^* S + i\gamma NS^* N + i2\gamma|S|^2 N + i2\gamma|N|^2 S, \quad (1)$$

$$\frac{\partial N}{\partial z} + i\frac{\beta_2}{2}\frac{\partial^2 N}{\partial t^2} + \frac{\alpha}{2}N = 0, \quad (2)$$

where we made the assumption that the linear nature of noise propagation is not affected by the signal. We validate this assumption by comparing the arrival times of the pulse center at the end of a transmission system, obtained directly from the NLSE and from Eqs. (1) and (2). Such a comparison shows that pulse-center position is given by the sum of three terms, one due to the noise-induced frequency shift, and other two related to signal-noise and noise-noise beatings at the photodetector. While the solution of the NLSE gives the total value, the term related to the pulse central frequency shift can be obtained from Eq. (1). Lumped amplification is not explicitly included in Eqs. (1) and (2), but its effects are considered by amplifying the signal and noise and adding the noise periodically.

In this study we focus on the evolution of pulse central frequency as a function of the launched peak power  $P_0$  and consider four different propagation regimes. In the soliton case, a lossless transmission medium is considered and the soliton regime is realized by launching a "sech" pulse satisfying the soliton condition,  $\gamma P_0 T_0^2/\beta_2 = 1$ , where  $P_0$  and  $T_0$  are the peak power and temporal width of the pulse, respectively. In the quasi-soliton case, attenuation is included along the line, and the peak power is increased such that we operate in the so-called average soliton regime. In the pseudo-linear case, we decrease the launched peak power substantially to prevent the formation of soliton-like pulses, but the nonlinear effects are not negligible. In the fourth case, we decrease the launched power further to operate very close to the linear regime; we name this regime quasi-linear. For each propagation regime, we consider two different situations: A single-span system, where a booster amplifier amplifies the pulse before it propagates over 100 km, and a five-span system with a total length of 500 km that results from the concatenation of five single-span systems. For each span, the amplifier gain of  $G = 20$  dB compensates fiber losses completely. Each amplifier also adds statistically

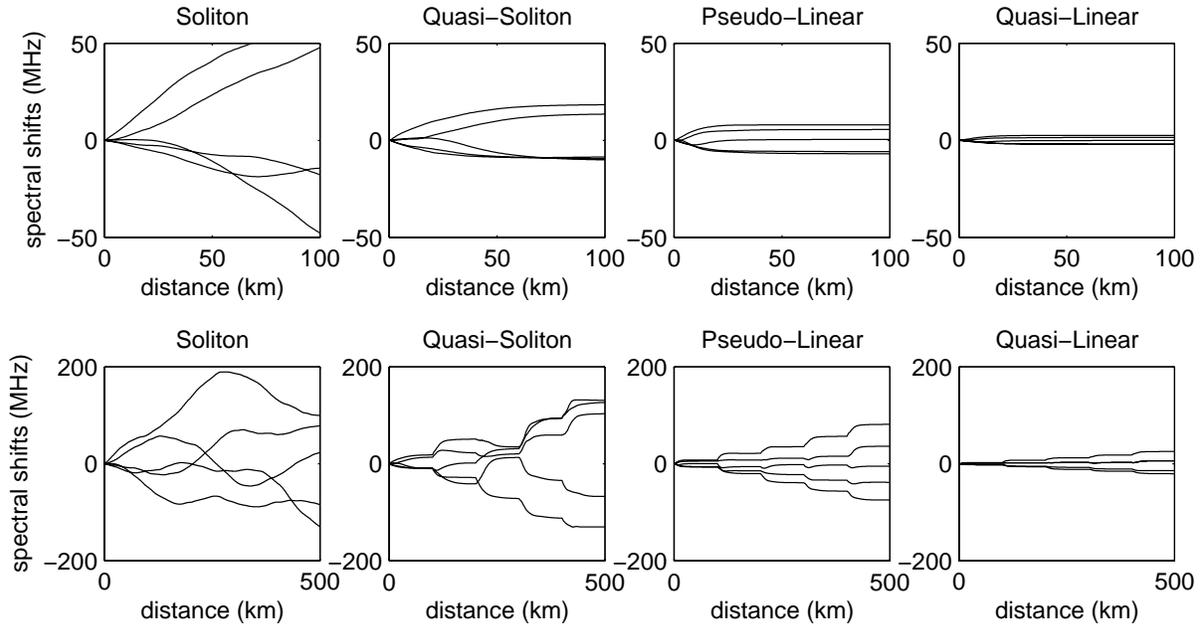


Fig. 1: Noise-induced spectral shifts in the case of a single 100-km span (top) and a five-span system (bottom) for four different system designs discussed in the text.

independent white Gaussian noise with the average power  $h\nu_0 n_{sp} B_{\text{opt}}(G-1)$ , where  $\nu_0$  is the carrier frequency,  $n_{sp}$  is the spontaneous-emission factor, and  $B_{\text{opt}}$  is the optical bandwidth.

Our results for both the single-span and five-span systems are shown in Figure 1. In the soliton case, we observe that frequency shift occurs along the whole span, in contrast with the other three cases where frequency shift ceases to occur after a certain distance. This is because the nonlinear effects remain strong for solitons during all the span with no losses and no pulse spreading. In the other cases, losses and dispersion-induced pulse spreading reduce the nonlinear effects, therefore, after some distance, frequency shift ceases to occur. This proves that the noise-induced frequency shift is a nonlinear effect, and no shift would occur in a pure linear regime. In the multi-span case, it is misleading to conclude from the preceding discussion that frequency shifts would be small in a pseudo-linear system. Indeed, Figure 1 shows that large shifts can occur for such systems due to a strong correlation between the signal and noise during propagation. The direction of frequency shifts is related to the phase difference between the signal and noise. As seen in Eq. (2), noise propagates in a linear fashion. On the other hand, the first term in the right side of Eq. (1) can vary considerably, depending on the propagation regime. This term is responsible for the signal self-phase modulation. If this term is large, as it is in the soliton case, frequency shifts will change directions frequently, preventing a continuous sliding of the central frequency. In the other three cases, self-phase modulation is less effective. As a result, signal and noise propagation are more correlated, in the sense that both signal and noise tend to propagate in a linear fashion, resulting in a continuous sliding of the pulse spectrum. Another interesting observation from Figure 1 is that the direction of the frequency sliding is mostly determined by the first amplifier in the line.

In conclusion, the buildup of the frequency shifts implies that this effect must be considered even for designing pseudo-linear systems. Otherwise, a large timing jitter would be observed at the receiver end.

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