A Probabilistic Model for the Demands on Link on Mesh Optical Networks

Abel R. P. Correia, Claunir Pavan, Armando Nolasco Pinto

Abstract— We present a probabilistic model for the number of demands on link on mesh optical networks. This model allows forecasting the expenses for capital equipment without requiring the complete dimensioning of the telecommunication infrastructure.

***Index Terms*— Demands on link, dimensioning, optical networks.**

I. Introduction

The dimensioning of optical networks is frequently addressed using extensive numerical processing. We aim to provide semi-analytical tools for the dimensioning problem. We expect to obtain less time consumption solutions and to gain a deep insight on the dimensioning problem.

Having a traffic demand, or an expected traffic demand, a telecommunication operator must set up a network to support that traffic minimizing the expenses for capital equipment.

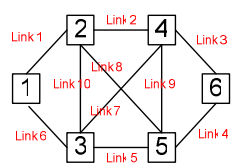
Traffic demands must be routed over the network. To do this a routing discipline must be followed. In this work a minimum hop routing discipline is assumed. Using this routing discipline, different solutions can be found for the routing problem. Each solution leads to a specific number of demands on link. What can we say about this specific number without knowing which routing was adopted? This is the main question that we intend to answer in this work.

II. Optical Mesh Network

A network is formed by a set of nodes and a set of links, where the nodes are connected through the links. A network topology can be modeled as a graph, G(N, L), where N is the number of nodes and L is the number of links.

A graph can be represented by an NxN adjacency matrix [g]. The matrix elements g_{ij} are either 1 or 0, respectively, whether a pair of nodes is directly connected or not. For instance, let's consider the network on fig.1 (a), with N=6 nodes and L=10 links and the adjacency matrix [g], see fig.1 (b).

The summation of all the values of [g] yields the number of one-way links, L_1, which is twice the number of two-way links, $L= L_1/2$. In this work we assume full-duplex links, therefore the [g] matrix is always symmetrical.



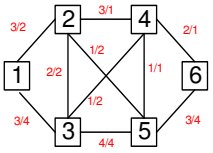
$$[g] = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

a) b)

Fig.1. A network topology, represented by a graph a) and by an adjacency matrix b).

The traffic can be represented by a demand matrix [d]. The [d] elements are integer numbers, representing the number of claims between nodes. The summation of all the [d] values gives the number of simplex-demands, D_1, which are twice the full-duplex demands, $D= D_1/2$.

$$[d] = \begin{bmatrix} 0 & 2 & 1 & 1 & 1 & 1 \\ 2 & 0 & 2 & 1 & 1 & 1 \\ 1 & 2 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$



a) b)

Fig.2. The demand matrix [d] a) and two valid routing solutions b) (solution 1/solution 2).

However given a network [g] and a demand [d], the number of demands on link is not univocally determined, because there are several solutions for the same routing problem. For instance the examples presented in fig.2 b) are two valid solutions, assuming a minimum hop discipline. Therefore a probabilistic model is necessary to describe the number of demands on link.

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Abel R. P. Correia is with Department of Electronic, Telecommunications and Informatics, University of Aveiro, 3810-193 Aveiro, Portugal, and with Institute of Telecommunications, 3810-193 Aveiro, Portugal; e-mail: Abel.Correia@vodafone.com.

Claunir Pavan, is with Institute of Telecommunications, 3810-193 Aveiro, Portugal

Armando Nolasco Pinto, is with Department of Electronic, Telecommunications and Informatics, University of Aveiro, 3810-193 Aveiro, Portugal, and with Institute of Telecommunications, 3810-193 Aveiro, Portugal, e-mail: anp@det.ua.pt.

III. Probabilistic Model

A) Approach I

Firstly, we obtained the number of different routings, R, using the routing matrix [r], which is an NxN matrix and each element r_{ij} has the number of routes – with minimum hops - between *i* and *j*. The matrix [r] can be obtained by the extended Dijkstra's algorithm.

For our network showed on fig.1, we have R=64, which can be obtained from the formula (1) and matrix [r], presented in fig. 3.

$$R = \prod_{d=1, i \neq j}^{D} d_{ij}.r_{ij} \qquad (1)$$

$$[\,r\,] = \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 4 \\ 1 & 0 & 1 & 1 & 1 & 2 \\ 1 & 1 & 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 & 1 & 1 \\ 2 & 1 & 1 & 1 & 0 & 1 \\ 4 & 2 & 2 & 1 & 1 & 0 \end{bmatrix}$$

Fig.3. The routing matrix [r].

Each routing can be represented by L one-dimensional vectors with D positions; each position, $x_{l,r,d}$, holds the value 1 if the demand is routed through the link and 0 otherwise.

In order to calculate the amount of demands that are carried by a link, we consider as W_l the random variable for the number of demands on link *l*. The number of demands for each routing *r* can be expressed as

$$w_{l,r} = \sum_{d=1}^{D} x_{l,r,d} \ , \qquad (2)$$

Considering all the R possible routings we can obtain the mean of W_l ,

$$\langle W_l \rangle \equiv \frac{1}{R}\sum_{r=1}^{R} w_{l,r} = \frac{1}{R}\sum_{r=1}^{R}\sum_{d=1}^{D} x_{l,r,d} \ . \qquad (3)$$

Thus, for our network example, we have $<W_1>=3.5$, and using the general formula of variance, we can reach the variance of the demands on link,

$$\sigma^2_{W_l} \equiv \frac{1}{R}\sum_{r=1}^{R}\left(w_{l,r} - \langle W_l \rangle\right)^2 = \frac{1}{R}\sum_{r=1}^{R} w^2_{l,r} - \langle W_l \rangle^2 \ , (4)$$

which for our case gives $\sigma^2_{W_1}$ =0.75.

B) Approach II

In this approach instead of the routing matrix [r] we are going to use the probability matrixes $[p^l]$, one matrix per link. Each element of the probability matrix, p^l_{ij}, gives the probability of using the link *l* to support one demand between node *i* and *j*. The elements of the probabilities matrixes can be obtained using a search algorithm. In our example for the link 1 we obtain:

$$[\,p^1\,] = \begin{bmatrix} 0 & 1 & 0 & ½ & ½ & ½ \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ ½ & 0 & 0 & 0 & 0 & 0 \\ ½ & 0 & 0 & 0 & 0 & 0 \\ ½ & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Fig.4. Matrix $[p^1]$. The link 1 connects node 1 with node 2.

Now, let's consider $p_{l,d}$ the probability of the demand *d* to be carried through link *l*. Starting from (3) we may write

$$\langle W_l \rangle = \sum_{d=1}^{D}\frac{1}{R}\sum_{r=1}^{R} x_{l,r,d} = \sum_{d=1}^{D} p_{l,d} \ . \qquad (5)$$

Using the elements of the $[p^l]$ and [d] matrixes we obtain

$$\langle W_l \rangle = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} d_{ij}.p^l_{ij} \ . \qquad (6)$$

Applying (6) to our example we obtain, as expected, $<W_1>=3.5$. Now we may obtain the variance of demands on link. Considering that the demands are independently routed and the expression for the variance of a binomial random variable, the corresponding variance $\sigma^2_{W_l}$ is obtained by

$$\sigma^2_{w_l} = \sum_{d=1}^{D}\sigma^2_{x_{l,r,d}} = \sum_{d=1}^{D} p_{l,r,d}(1 - p_{l,r,d}) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}(1 - p^l_{ij})^2 .(7)$$

Considering our example we obtain $\sigma^2_{W_1}$ =0.75, as expected.

IV. Considerations

In this paper, we present a model, based on two distinct approaches, for the number of the demands on link on optical mesh-networks. We have applied both approaches to a small network and we have shown that the results are valid and the results can be obtained with a few inputs, the first approach depends on [g], [d] and [r] matrixes, while the second approach exempt the [r] and introduce the $[p^l]$ matrix, maintaining the dependency on [g] and [d] matrixes. In the preliminary tests that we have done for larger networks we noticed that the second approach is computationally faster, as it does not require the R routings.

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References

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