Geometric Interpretation of Waveplate-Induced Polarization Transformation in Stokes Space

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***Abstract*— A Stokes space geometrical interpretation of waveplate-induced state of polarization (SOP) transformation is presented. We show that there exist different ways to transform between two any SOPs.**

***Index Terms*— Rotation matrices, wave plates, polarization controllers (PC), Stokes space.**

I. INTRODUCTION

POLARIZATION control and monitoring are major issues in modern high-speed ($\geq$ 40 Gb/s) and long-haul ($\geq$ 100 km) optical communication systems, due to the small tolerance of these systems to signal distortion. By other side, a device with the ability to produce a uniform scatter over the Poincaré sphere can be useful in the development of polarization mode dispersion (PMD) emulators and in PMD compensation. Indeed, some authors have proposed PMD emulators, based on pieces of polarization maintaining fibers interconnected with uniform scattering devices [1], [2]. The fiber-coil based polarization controller [3] can be used to scatter the state of polarization (SOP) over the Poincar sphere. In this work a geometrical interpretation of waveplate- and polarization controllers-induced SOP transformation in Stokes space is presented.

II. EXAMPLES OF POINCARÉ SPHERE REPRESENTATION

In the Stokes space the quarter and half waveplates are, respectively, represented by the following rotation matrices [4]

$$\mathbf{M}_{\lambda/4}(\theta) = \begin{bmatrix} \cos^2(2\theta) & \cos(2\theta)\sin(2\theta) & \sin(2\theta) \\ \cos(2\theta)\sin(2\theta) & \sin^2(2\theta) & -\cos(2\theta) \\ -\sin(2\theta) & \cos(2\theta) & 0 \end{bmatrix} \quad (1)$$

and

$$\mathbf{M}_{\lambda/2}(\theta) = \begin{bmatrix} 2\cos^2(2\theta)-1 & 2\cos(2\theta)\sin(2\theta) & 0 \\ 2\cos(2\theta)\sin(2\theta) & -2\cos^2(2\theta)+1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad (2)$$

where θ represents the orientation of the principal axes. Nevertheless, these two matrices may be interpreted geometrically as a rigid rotation of a certain angle about a unit vector. A general rotation matrix, $\mathbf{M}$, can be expressed in the following vectorial form [4]–[6]

$$\mathbf{M} = \cos(\varphi)\mathbf{I} + [1-\cos(\varphi)](\hat{r}\hat{r}\cdot) - \sin(\varphi)(\hat{r}\times), \quad (3)$$

where $\mathbf{I}$ is the identity matrix, $\hat{r}$ and φ represent the axis and the angle rotation, respectively, $(\hat{r}\hat{r}\cdot)$ is the projection operator and $(\hat{r}\times)$ is the crossproduct operator. The rotation angle and the rotation axis are extracted from $\mathbf{M}$ [5], [6],

$$\cos(\varphi) = \frac{1}{2}\left(Tr(\mathbf{M}) - 1\right), \quad (4)$$

and

$$r_1 = \frac{1}{2\sin(\varphi)}[\mathbf{M}_{32} - \mathbf{M}_{23}], \quad (5)$$

$$r_2 = \frac{1}{2\sin(\varphi)}[\mathbf{M}_{13} - \mathbf{M}_{31}], \quad (6)$$

$$r_3 = \frac{1}{2\sin(\varphi)}[\mathbf{M}_{21} - \mathbf{M}_{12}], \quad (7)$$

where Tr is the matrix trace, r_i are the components of $\hat{r}$ and $\mathbf{M}_{ij}$ are the elements of $\mathbf{M}$. The foregoing formulas may be used to calculate the rotation angles and the rotation axes for the quarter and half waveplates. Rotation angles are $\pi/2$ and π, respectively, i.e. the rotation angle coincides with the phase delay induced by waveplates. By other side, both waveplates present the same θ-dependent rotation axis,

$$\hat{r}(\theta) = \begin{bmatrix} \cos(2\theta) \\ \sin(2\theta) \\ 0 \end{bmatrix}. \quad (8)$$

Note that, when represented in the Poincaré sphere, $\hat{r}$ lies along the equator, like linear states of polarization.

In Fig. 1 are represented the SOP transformations induced by a quarter and half waveplates, placed at $\theta = 45°$, to an initial SOP $\hat{s}_i = [-1, 0, 0]^T$, where T is the transpose. The

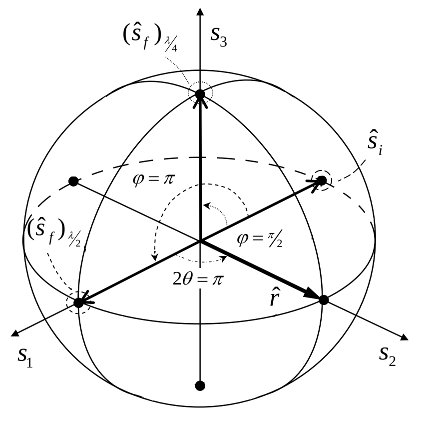

Fig. 1. SOP transformations induced by a quarter and half waveplates, placed at $\theta = 45°$, to an initial SOP $\hat{s}_i = [-1, 0, 0]^T$.

SOP $\hat{s}_i$ is, in both cases, rotated around the vector $\hat{r}$: the final SOP is $(\hat{s}_f)_{\lambda/4} = [0, 0, 1]^T$ and $(\hat{s}_f)_{\lambda/2} = [1, 0, 0]^T$ to the quarter and half waveplates, respectively.

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III. Quarter-Half-Quarter waveplate Polarization Controllers

In general, fiber-coil based polarization controllers are constructed by the concatenation of one half waveplate and two quarter waveplates (the first one is placed between the last two). The analytical waveplate angles expressions, in order to transform an arbitrary input SOP into an also arbitrary output SOP, was already presented in literature [1]. In that work it is assumed that the first quarter waveplate converts the input SOP, $\hat{s}_i$, into a linear SOP, $\hat{s}_j$, the half waveplate changes between any two linear SOPs and the second quarter waveplate converts a linear SOP, $\hat{s}_k$, into the desired output SOP, $\hat{s}_o$.

In order to visualize the different transformations induced by polarization controllers, we have choose an initial SOP $\hat{s}_i = [0, \sqrt{1/2}, \sqrt{1/2}]^T$ and three random values to the three waveplates angles. Afterwards other three angles were analytical calculated, using the expressions presented in [1], for the initial SOP $\hat{s}_i$ and the obtained output SOP, $\hat{s}_o = [0.8779, -0.0542, 0.4757]^T$. Fig. 2 shows the two different

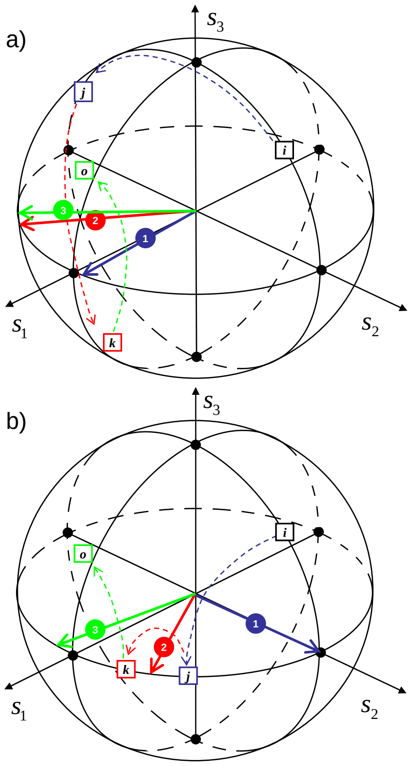

Fig. 2. State of polarization transformation steps induced by a polarization controller: a) $\hat{s}_o$ was obtained from $\hat{s}_i$ and three random waveplate angles; b) the path was obtained using $\hat{s}_i$ and $\hat{s}_o$ as input and output SOP into the analytical waveplate angles model [1]. Circles represent the rotation axis for the three waveplates and boxes the SOPs (only subscripts are used).

paths obtained to transform $\hat{s}_i$ into $\hat{s}_o$: the first, Fig. 2a), corresponds to a random waveplates angles set and the second, Fig. 2b), corresponds to the analytical calculated waveplates angles. From this example we can say that the input-linear-linear-output SOP path method is more simple when compared to others paths.

A more exhaustive comparison of these two different processes, using the same procedure and others input SOPs, was performed. Fig. 3 shows the random (○) and calculated angles (●), for the three waveplates, respecting to a particular input SOP $\hat{s}_i = [0, 0, 1]^T$. Note that, to this input SOP, the calculated angles for the first waveplate is always the same and the calculated angles for the third waveplate coincides with the random angles.

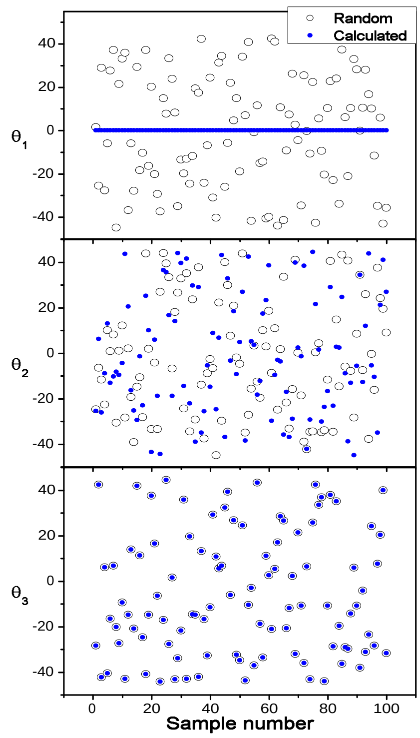

Fig. 3. Random (○) and calculated (●) waveplate angles: θ_1 are the first quarter waveplate angles, θ_2 the half waveplate angles and θ_3 the second quarter waveplate angles.

IV. Conclusion

We have presented in this paper a geometric analysis of the SOP transformations induced by wave plates. Results show that there are different ways to transform between an initial and a final SOP. The analytical method seems to use simple intermediate paths.

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