A GRAPH PROBLEM IN THE CONTEXT OF OPTICAL NETWORKS DIMENSIONING

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Abstract – The dimensioning of a multi-layer optical network is a complex problem, frequently addressed using extensive numerical processing. We aim to provide semi-analytical tools for the dimensioning problem. We expect to obtain less time consumption solutions and to gain a deep insight on the dimensioning problem. In this paper, we present a set of three sub-problems on combinatorial and probabilities with applications on design of multi-layer optical networks. The first sub-problem is the generation of all combinations of topologies, given a number of nodes and links, where there is connectivity among all the nodes; the second sub-problem is the determination of the amount of different average spans for all valid topologies; the third sub-problem is the calculation of the probability of each average value.

Keywords – Optical networks, dimensioning, graphs theory, combinatorial, probabilities.

I. INTRODUCTION

The dimensioning of optical networks are a very active research area due to the grown of bandwidth demand and the new functionalities that the optical layer can provide [1-3].

In the dimensioning problem the bandwidth demands must be satisfied. Besides that, expenses with capital equipment (CAPEX), network operation (OPEX) and network management (MANEX) must be minimized.

An optical network multi-layer is characterized by allowing routing in the optical layer. The topology is a set of paths which support the traffic demands. The determination of network architecture for multi-layer optical networks and theirs costs can be done using applications programs that support the analysis. The analysis must be done carefully while minimizing the cost and satisfying some objective functions [4].

The problem of the choice of a topology for a network within several possibilities can be solved by using either an exact or heuristic algorithm. The first is usually only used for small networks because the complexity and demand for computational capacity increases exponentially with the number of nodes in the network. The second does not make the optimal solution, but it is usually simpler and faster.

We understood that semi-analytical methods are the most appropriate way for the treatment of large network cases, because it can bring results in smaller time than the numerical methods. The analytical results are of addition importance by the fact that they can provide a deep insight in the problem. Frequently, this forms the basis for decision making. Unreliable results bring a risk of incorrect decisions and may lead to high costs.

Our goal regarding the utility and character of the problem has been to permit results to be computed very fast with useful accuracy for a very wide range of networks sizes and thereby to provide valuable understanding and guidance.

In this paper, we formulated three sub-problems of the dimensioning problem in optical networks; we believe that they can stimulate researchers to present analytical solutions for the questions.

In section II, we formulate mathematically the problems. In section III and IV, we present, respectively, a simple and a more complex case. Finally, in sector V, we conclude the paper.

II. FORMULATION OF THE PROBLEM

A network is formed by a set of nodes and a set of links; the nodes are connected through the links. Starting from here we are going to use the nomenclature of the mathematicians to designate the elements of the problems, the word vertexes means nodes and its number is denoted by N, the word edges means full-duplex links and its number is denoted by L, and the word graph means topology and it is denoted by G(N,L).

The graph can also be represented by a matrix [g]. The matrix elements g_{ij} are either 0 or 1 in value and specify whether a pair of vertexes is connected via a physical edge. The summation of all the values of the matrix elements yields the number of one-way edges, which is twice the number of two-way edges, L.

The definition of a graph to our problem goes essentially by some different moments, one of them is the determination of the number of vertexes and the number of edges that the graph will have, and other moment is to choose the way to connect the vertexes.

The focus here starting from the second moment, when we already have the information of the amount of vertexes and edges that we can apply on the graph.

One graph need to be complete. A graph disconnected represents an invalid option. This is our first question: how many valid graphs can we get for a given N and L?

Each one of these graphs may have a different mean number of spans ‹s›; the number of spans between a pair of vertexes is defined as the minimum number of edges that a claim traverses between the origin and destination vertexes pair. Algorithms for determining the minimum number of spans s_{ij} between vertexes pair (i,j) from the matrix representing the graph G(N,L), [g], can be used, and so, [s] and ‹s› may be computed, the matrix [s] represents the exact number of spans between each pair (i,j) and a mean number of spans ‹s› can be calculated by,

$$\langle s \rangle = \frac{1}{2C}\sum_{i=1}^{N}\sum_{j=1}^{N} s_{ij} \, , \qquad (1)$$

where s_{ij} means the amount of spans – edges - between i and j, and it will be obtained from matrix [s], where C means the maximum number of claims, which it can be calculated by,

$$C = \frac{(N-1)N}{2} \, . \qquad (2)$$

The maximum number of claims C is directly related to the maximum number of claims that each vertex can do.

Then, our second question is: given N vertexes and L edges, how many different values of mean number of spans will be possible?

If we divide the number of times that an average appeared by the sum of the number of appearing of all averages, we obtain the probability of this average.

Thus, we can formulate our third question: what is the probability of each average value?

III. A SIMPLE CASE

For this simple case, we assumed a simple graph with N=4 vertexes and L=3 edges.

Initially we go to discovery how many combinations there are with N=4 and L=3. For this, we go use [5],

$$c(n,k) = \frac{(n)!}{(k)!(n-k)!} \, . \qquad (3)$$

This indicates that n is the maximum number of edges from which we want choose, and it is calculated by n=N(N-1)/2 and k is the number of edges to be chosen to generate combinations.

The minimum value for k is k=N-1, this value represents the minimum amount of edges that connect all vertexes. And n represents the maximum amount of edges, when all vertexes have degree of N-1.

Considering N=4 and L=3, using (3), we have six edges as maximum and wish to choose three (L=3) for our combinations, then, we have 6!/3!(6-3)!=20 ways, as in Fig. 1.

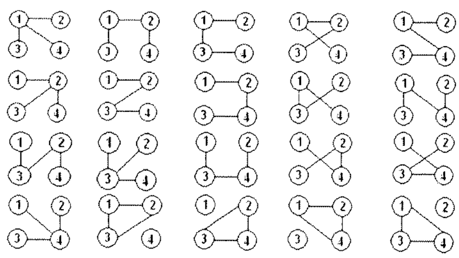

Fig. 1. All possibilities of combination for a graph with N=4 vertexes and L=3 edges. We enumerate the different solutions starting from the upper left and going row by row.

As we can see, there are some graphs that we should discard because they are not complete graphs, (the solution 17 to 20 on Fig. 1). Then, for our first sub-problem, in this case, we have 16 as response.

We obtained the values of invalid and valid combinations through a computational verification which consisted on verifying if each vertex had at least an edge and if each vertex had a road with any other vertex.

Now they remain 16 valid graphs, and we chose solution 6 for our next sub-problem, this graph is also represented by matrix [g], see table 1.

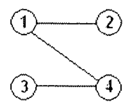

Fig. 2. A possible graph for a network with N=4 vertexes and L=3 edges.

Table 1

[g]	*1*	*2*	*3*	*4*
1	0	1	0	1
2	1	0	0	0
3	0	0	0	1
4	1	0	1	0

Starting the second sub-problem solution, we need to calculate the mean number of spans for this combination, which can be obtained after having matrix [s] by (1) and (2).

In this case, the spans matrix [s] would be like table 2, and was obtained looking for the shortest path for each vertexes pair.

Table 2

[s]	*1*	*2*	*3*	*4*
1	0	1	2	1
2	1	0	3	2
3	2	3	0	1
4	1	2	1	0

The mean spans resulting from (1) and (2) for solution 6 is ‹s›=1.67.

If we repeat this for all the 16 valid graphs, we will have only 2 different values of mean spans, ‹s›=1.5 and ‹s›=1.67. Therefore, 2 is the answer for our second sub-problem.

Now that we already have the numerical solution for our two first sub-problem, we want to know what is the probability of each ‹s›.

We denote $P_{‹s›}$ as a probability of ‹s› and we have verify that the value ‹s›=1.5 appeared 4 times in a total of 16 and the value ‹s›=1.67 appeared 12 times in a total of 16, then the probability of ‹s›=1.5 is $P_{1.5}$ = 4/16 = 0.25 and the probability of ‹s›=1.67 is $P_{1.67}$ = 12/16 = 0.75. This represented the solution of the third sub-problem.

The results of the two previous problems can also be seen in the Fig.3.

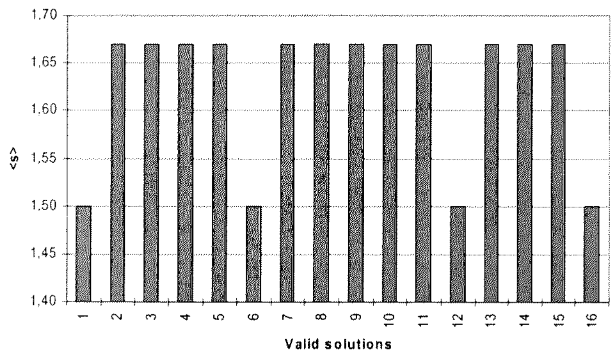

Fig. 3. In the xx′ axis are represented the 16 valid graph obtained in sub-problem 1 and in the yy′ axis are represented the average number of spans.

In Fig. 4, it is shown the results of the three sub-problems; valid solutions, amount of ‹s› and probability of each ‹s›.

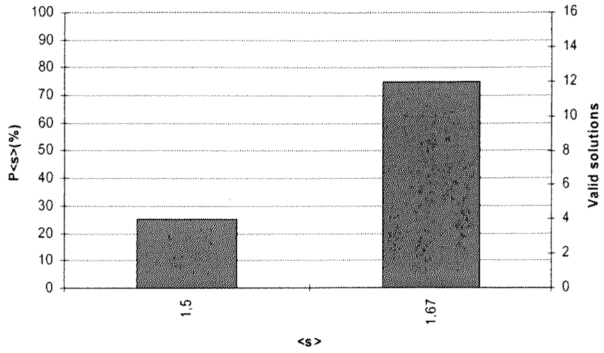

Fig. 4. Probability of each ‹s› with N=4 and L=3, number of valid solutions and number of different ‹s›.

The time for a computer with processor Pentium IV 3.2GHz, 1GB of Random Access Memory (RAM) to solve these three sub-problems was less than one second.

IV. A MORE COMPLEX CASE

Now, we assumed a graph with N=7 vertex and L=8 edges, the number of combinations by using (3) is 203490, but there are 46935 invalid combinations, therefore we get 156555 valid graphs.

The graph presented in Fig. 3 was chosen by chance of the group of valid solutions for this case.

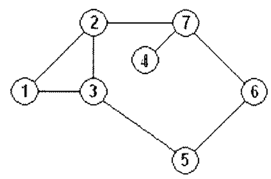

Fig. 4. A possible graph for a network with N=7 vertex and L=8 edges.

The matrix of connections [g] and matrix of spans [s] are represented, respectively, in table 3 and table 4.

Table 3

[g]	*1*	*2*	*3*	*4*	*5*	*6*	*7*
1	0	1	1	0	0	0	0
2	1	0	1	0	0	0	1
3	1	1	0	0	1	0	0
4	0	0	0	0	0	0	1
5	0	0	1	0	0	1	0
6	0	0	0	0	1	0	1
7	0	1	0	1	0	1	0

Table 4

[s]	*1*	*2*	*3*	*4*	*5*	*6*	*7*
1	0	1	1	3	2	3	2
2	1	0	1	2	2	2	1
3	1	1	0	3	1	2	2
4	3	2	3	0	3	2	1
5	2	2	1	3	0	1	2
6	3	2	2	2	1	0	1
7	2	1	2	1	2	1	0

The mean spans for this sample, using (1) and (2) is ‹s›=1.81, but if we repeat that for all 156555 graphs, we have only 13 different values for ‹s›.

In Fig.5, we present a graphic that show the solutions for the three sub-problems; valid solutions, amount of ‹s› and probability of each ‹s›.

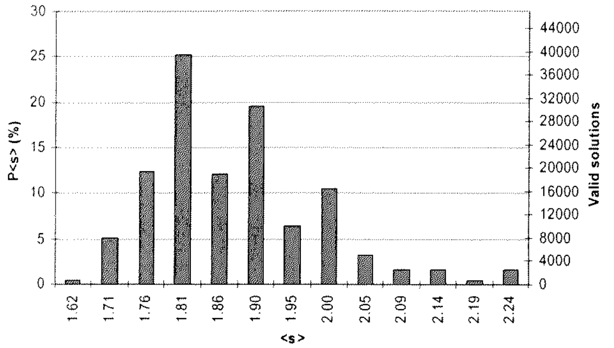

Fig. 5. Amount and probability of ‹s› and amount of valid solutions with N=7 and L=8.

The time for a computer with processor Pentium IV 3.2GHz, 1GB of Random Access Memory (RAM) to process these operations was greater than 43 minutes.

V. DISCUSSION

In this paper, we consider the problem of dimensioning in multi-layer optical networks and we present three sub-problems that we expect can be resolved analytically. This will be a valuable achievement due to the demand of time and power computation needed to obtain a solution through numerical methods.

As we can see the problem enlarges quickly. We presented here two examples, which were treated computationally, the time demand for obtained the solution for the case with 4 vertexes and 3 edges was smaller than one second and the time demand for the case with 7 vertexes and 8 edges was greater than 43 minutes. We can notice that the exponential growth soon turns difficult the numerical processing.

Examples with small graphs or graphs in which the number of links are near the limit may be processed numerically and with a modest time of computing process, but on practice, we want to treat large graphs as the European Optical Network that has 20 vertexes and 38 edges [7] or the USA Optical Network that has 100 vertexes and 171 edges [6]. These cases, with numerical techniques, would be computationally impracticable.

AKNOWLEDGEMENTS

This work was partially supported by the Portuguese Scientific Foundation FCT, through the "IP over WDM Networks" project (POSI/EEA-CPS/59566/2004), FEDER and POSI programs.

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