Broadband Superhighway
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Timing jitter statistics due to soliton interaction and Gordon-Haus effect

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Summary

Semi-analytical bit-error-rate (BER) evaluation methods rely on the knowledge of the optical pulse arrival time statistics. In a ligthwave communication system based on solitons, mutual interactions among pulses and spontaneous emission noise, due to the amplification process during propagation, lead to non-Gaussian statistics. In this contribution, we derive an analytical model for the arrival-time statistics in a system propagating a random sequence of solitons by considering the interaction forces among solitons. In a soliton system, dominated by the soliton interaction forces, the noise can be considered a small perturbation. By using the analysis of Gordon and Haus, we present an approximate expression for the arrival-time statistics taking into account both the soliton-soliton interaction and the amplified noise effect. The analytical results are in good agreement with those obtained by numerical simulations.

Introduction

Full Monte-Carlo simulations of long-haul soliton systems with a BER less than 10^{-9} require huge amounts of computing time even if powerful resources are used. So, it is crucial to find alternative and efficient ways of determining with accuracy the BER. To this end, several semi-analytical techniques have been presented based on the Chernoff bound, modified Chernoff bound, Gaussian approximation, and saddlepoint approximation [1, 2]. These methods use the moment generation function and the statistics of the arriving time of optical pulses. They have been applied to systems based on lumped amplifiers, where the jitter is due to the amplified spontaneous emission (ASE) noise. In this case, we can assume that the amplifier noise is Gaussian, and it can be shown that the jitter is also Gaussian. However, it was pointed out recently by Menyuk (1995) and by Georges (1995) that the soliton interaction modifies these statistics [3, 4]. The analytical characterization of the arrival-time statistics of a random sequence of solitons due to the interaction between adjacent solitons is the main goal of this work. This knowledge allow us to derive an approximation of the arrival time statistic in a more realistic situation, taking into account both the soliton interaction and the ASE noise.

Two-soliton system

The propagation of pulses in an optical fiber can be described by the nonlinear Schrödinger equation (NLSE) which is derived from the Maxwell equations [5]. To study multiple-pulse systems, we start with the NLSE for the case of two solitons. An approximate solution for the case of two solitons based on the quasi-particle approach was presented by Karpman and Solov'ev [6]. Another approximation, derived directly from the exact two-soliton equation, was derived by Gordon [7]:

$$u(\tau,\zeta) = \exp(i\Omega)\left\{A_1 \operatorname{sec}h\left[A_1(\tau - q)\right]\exp(i\theta_1) + A_2 \operatorname{sec}h\left[A_2(\tau + q)\right]\exp(i\theta_2)\right\} , \quad (1)$$

Expression (1) describes the evolution of a pair of solitons inside an optical fiber. At each point inside the fiber the separation between the solitons is 2q, the amplitudes of the two solitons are A_1 and A_2, and the relative phase difference is θ_2-θ_1. For simplicity of notation we introduce $\psi = (\theta_2-\theta_1)/2$.

The separation between the soliton, 2q, and their phase difference, 2ψ, are functions of the distance of propagation, ζ, satisfying:

$$\rho\exp(q + i\psi) = 2\cosh(\zeta_o + i\rho\zeta) \quad (2)$$

where ρ and ζ_o are constants of the propagation determined by the initial separation, phase difference and its first derivative. The first and second space derivatives of equation (2) combine to yield the equations of motion:

$$\frac{\partial^2 q}{\partial\zeta^2} = -4\exp(-2q)\cos(2\psi) \ , \quad (3)$$

$$\frac{\partial^2 \psi}{\partial\zeta^2} = 4\exp(-2q)\sin(2\psi) \ . \quad (4)$$

Equations (3) and (4) show that the dynamics of the soliton pair is due entirely to interaction forces that depend exponentially on their separation and sinusoidally on their relative phase. If q_o and ψ_o are the initial separation and phase difference respectively, we obtain the followings expressions for q and ψ during the propagation:

$$q(\zeta) = q_o + \frac{1}{2}\ln\left[\frac{\cosh(4\exp(-q_o)\sin(\psi_o)\zeta) + \cos(4\exp(-q_o)\cos(\psi_o)\zeta)}{2}\right] , \quad (5)$$

$$\psi(\zeta) = \psi_o + \frac{1}{2i}\ln\left[\frac{\cos(2\exp(-q_o)\exp(-i\psi_o)\zeta)}{\cos(2\exp(-q_o)\exp(i\psi_o)\zeta)}\right] . \quad (6)$$

Since a zero phase difference between neighboring solitons leads to the worst case, we consider this case only. Choosing $\psi_o = 0$, in eq. (5), we obtain the following expression for the separation between the two solitons:

$$q = q_o + \ln[\cos(a\zeta)] , \qquad (7)$$

where $a = 2\exp(-q_o)$.

Three-soliton system

To extend the above results to the three-soliton case we note first that the force between adjacent solitons depends on their separation and relative phase. Then in a system of three solitons, where the distance and relative phase between a side soliton (A and C in Fig. 1) and the middle soliton (B) are the same, the interaction of solitons A and C relative to B are of opposing signs. Because of these balancing forces, the middle soliton is fixed.

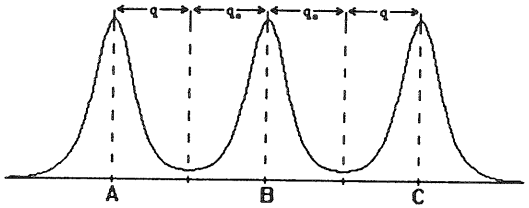

Fig. 1. In the three-soliton system in phase, the interactions of solitons A and C relative to B are of opposing signs.

In the two-soliton case the separation is equal to 2q. However in the three-soliton case, because the middle soliton is fixed, the separation between the side soliton and the middle one is q_o+q, where q_o is the initial separation. If we use this fact in equation (3) and (4), we obtain the following pair of equations for the three-soliton dynamics:

$$\frac{\partial^2 q}{\partial \zeta^2} = -2a\exp(-q)\cos(2\psi) , \qquad (8)$$

$$\frac{\partial^2 \psi}{\partial \zeta^2} = 2a\exp(-q)\sin(2\psi) , \qquad (9)$$

where $a=2\exp(-q_o)$.

The solution of equations (8) and (9), in the case of in-phase pulses provides:

$$q = q_o + \ln\left[\cos^2\left(\frac{1}{\sqrt{2}} a\,\zeta\right)\right] . \qquad (10)$$

Comparing (10) with the corresponding solution (7) for the two-soliton case, we see that besides the square in the cosine function, in the case of three solitons the period is $\sqrt{2}$ times the period of the two-soliton system. This mean that the soliton interaction is weaker for the case of three solitons compared with the case of two solitons because of the presence of the third soliton.

To test the validity of our analytical result, we performed a simulation by solving numerically the NLSE for the case of two and three solitons. The results are shown in Fig. 2. As we can see, the numerical results are in agreement with the analytical ones.

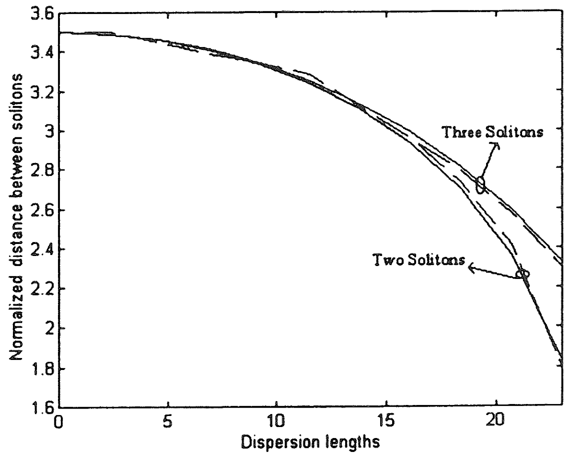

Fig. 2. Normalized distance between solitons, the solid line is the analytical solution, the dashed line is obtained by numerical solution of the NLSE.

Infinite number of solitons

To generalize our results, we start with the four-solitons case in which the two middle solitons (B and C in Fig. 3) are practically fixed because each one is surrounded by neighbors pulses exerting opposing forces. So with a good degree of approximation we can say that the two middle solitons are fixed and the side solitons (A and D) behave like in the case of three-soliton system.

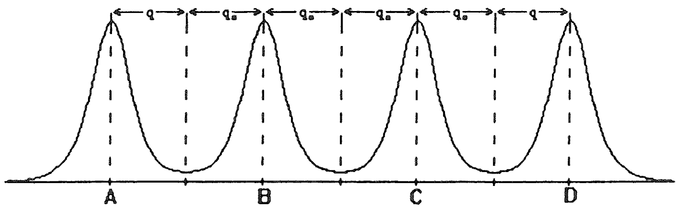

Fig. 3. In the four-soliton system the middle solitons (B and C) are practically fixed because each one is surrounded by neighbors pulses exerting opposing forces.

It is obvious that we can extend this approximation for the case of more than four solitons. In a long sequence of pulses the only ones where the interaction forces are important is the first and the last one. So the case of more than three solitons can be reduced to the three-soliton system.

In a lightwave communication system making use of pulse-code modulation the pulse sequence is virtually random, containing long sequences of pulses but also isolated and pair of pulses. To analyze the soliton interaction in a long sequence of pulses, we can divide the sequence in small slices and reduce each slice to one of the cases presented above. In effect, the presence of one soliton only disturbs its neighbors, because the interaction forces decrease exponentially with the separation. In Fig. 4 we have a sequence of 12 bits that we can reduce to the cases of isolated pulse (H), two-soliton case (A and B, K and L) and three-soliton system (D and E and F).

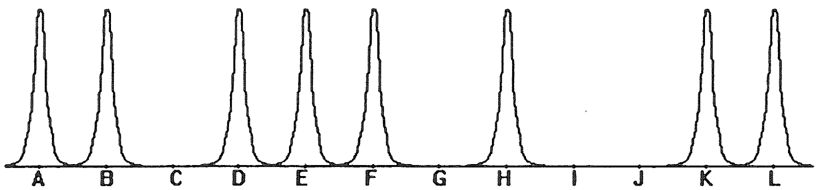

Fig. 4. To analyze the soliton interaction in a long sequence of pulses we can divide the sequence in small slices and analyze each slice.

In order to determine analytically the arrival time statistics we consider all possible sequences of 5 bits giving a total of 160 bits, out of which 80 bits contain pulses, assuming that "1" and "0" bits are equally likely to occur. It

can be shown that 40 pulses do not suffer any deviation, 20 pulses have the deviation given by expression (7) and another 20 have the deviation given by (10). So, the statistics of the arriving times is a bar graph, with 50% weight for zero time deviation, 25% corresponding to a deviation equal of the two-soliton system and 25% with deviation equal of the three-soliton system as shown in Fig. 5. These results were confirmed by solving the NLSE numerically.

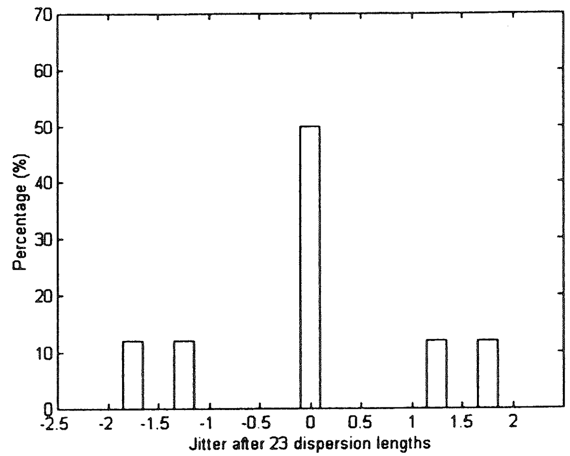

Fig. 5. Arrival time statistic of a pseudo-random sequence, after the propagation of 23 dispersion lengths. We can use expressions (7) and (10) to determine the side lobes positions.

The amplified spontaneous emission noise

In a real optical communication system based on optical solitons fiber losses must be compensated by optical amplifiers which introduce spontaneous emission noise as an unavoidable product of the amplification process. The superposition of spontaneous emission noise on a train of soliton pulses produces a random change of the soliton frequency that causes a change of the group velocity of individual solitons, which translates into a random jitter at the receiver. This phenomenon is know as the Gordon-Haus effect [8]. The standard deviation of the arrival time of the pulses can be computed from the Gordon-Haus formula [9],

$$\sigma = \left[\frac{1.763 N_{sp} N_2 Dh(G-1)L^3}{9 t_s A_{eff} L_{amp} Q} \right]^{1/2}, \qquad (11)$$

where N_{sp} is the spontaneous emission factor, N_2 is the nonlinear constant of the fiber, D is the first-order group velocity dispersion of the fiber, h is the Planck's constant, G is the amplifier gain, L is the total length of the link, A_{eff} is the effective mode area, L_{amp} is the amplifiers spacing and Q is the power enhanced factor [9].

The spontaneous emission noise acts together with the soliton interaction during the pulse propagation. Although the interaction forces between solitons are phase and amplitude sensitive, in a system with low noise we can assume that the noise only produces a small change in the statistic time distribution presented in fig. 5. In this case the arrival time statistic can be better characterized by a five Gaussian lobe function given by,

$$M(s)=\frac{1}{8\sqrt{2\pi}\sigma}\exp\left[-\frac{(s-s_{2-})^2}{2\sigma^2}\right]+\frac{1}{8\sqrt{2\pi}\sigma}\exp\left[-\frac{(s-s_{3-})^2}{2\sigma^2}\right]+\frac{1}{2\sqrt{2\pi}\sigma}\exp\left[-\frac{s^2}{2\sigma^2}\right]+\frac{1}{8\sqrt{2\pi}\sigma}\exp\left[-\frac{(s-s_{3+})^2}{2\sigma^2}\right]+\frac{1}{8\sqrt{2\pi}\sigma}\exp\left[-\frac{(s-s_{2+})^2}{2\sigma^2}\right], \quad (12)$$

where σ is given by expression (11) and s_{2-}, s_{2+} and s_{3-}, s_{3+} are respectively given by expression (7) and (10).

We performed a numerical simulation to show the validity of our approximation, Fig. 6 shows the arrival time distribution for a 5000km long system with 200 lumped amplifiers working at 5 Gbit/s.

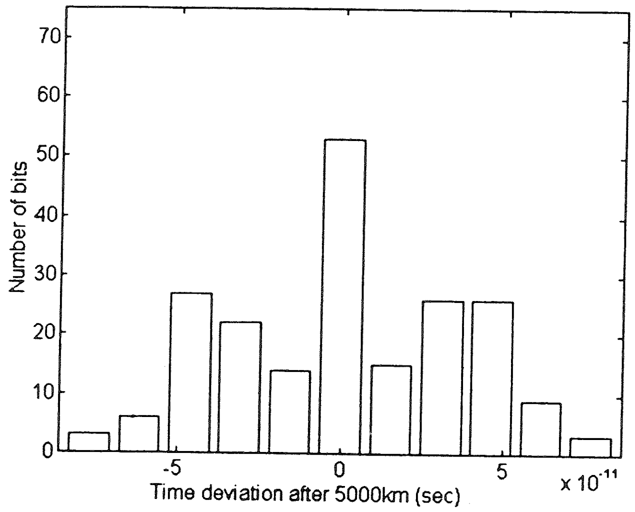

Fig. 6. Arrival time statistic of a pseudo-random sequence, after 5000 km propagation.

Conclusions

We have derived analytical expressions for the arrival times of multiple-soliton systems dominated by soliton interaction forces. Furthermore, it was shown that the arrival times statistics follow a multi-bar graph with weight 0.5 for zero time deviation, 0.25 corresponding to time deviation given by expression (7) and 0.25 time deviation given by expression (10). Simulations have confirmed the statistics obtained analytically. In a system with low noise we can use still use expression (7) and (10) in conjugation with expression (11) to obtain an approximation for the arrival time statistics.

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